

LYAPUNOV-STABLE ASSIST-AS-NEEDED CONTROL FOR HUMAN-EXOSKELETON INTERACTION IN POST STROKE REHABILITATION**Aybars Oztuna****ORCID ID - 0000-0003-4434-9792****<https://orcid.org/0000-0003-4434-9792>**Researcher, School of Computing, Mathematics and Physics, University of Portsmouth, UK
Fellow, Swedish Institute**ABSTRACT**

Hemiparesis occurs in more than 60 percent of stroke survivors on the six-month follow-up, and the subacute window is the most optimal neuroplastic recovery interval. Current powered exoskeleton systems provide deterministic assistance schedules with no formal closed-loop stability assurances in the presence of the time-varying neuromuscular perturbations of stroke groups. It is described in the present paper as a Lyapunov-stable Assist-as-Needed (AAN) impedance controller of bilateral lower-limb exoskeleton rehabilitation, and whose ultimate boundedness (UUB) of joint tracking error under constrained human motor disturbances is shown to be uniform. Surface EMG-driven effort estimation modulates an assistance factor $\alpha(t) \in [0,1]$ via a passivity-consistent adaptive law with guaranteed boundedness. A prospective, single-blind randomized controlled trial (NCT05841732; N=40 subacute stroke participants; n=20 per arm) validated the framework over six weeks. The experimental group achieved Fugl-Meyer Lower Extremity gains of $\Delta\text{FMA-LE} = 9.4 \pm 2.8$ versus 5.1 ± 3.2 in controls ($F(1,38) = 22.7$, $p < 0.001$, partial $\eta^2 = 0.37$, Cohen's $d = 1.42$). Lyapunov function monotonicity was confirmed in 94.7% of 360 therapy sessions. The assistance factor $\alpha(t)$ declined from 1.0 to 0.38 ± 0.14 over six weeks, correlating strongly with FMA-LE gain ($r = -0.74$, $p < 0.001$), establishing $\alpha(t)$ as a continuous neuromotor recovery biomarker. These results confirm formal closed-loop stability as outcome-relevant and clinically provable criteria of patient-adaptive rehabilitation exoskeletons.

Keywords:

Rehabilitation robotics, Lyapunov function, post-stroke rehabilitation, human-exoskeleton interaction

1. INTRODUCTION

Every year, stroke impacts up to 13.7 million people, and the hemiparesis of lower limbs and the severe impairment of autonomy remain in most stroke survivors (Stinear et al., 2020). The subacute recovery window, which is between two weeks and six months after ictus, is the most neuroplastic period of time when there is the most engagement of experience-dependent cortical reorganization (Nam et al., 2019). More than 400-600 repetitions of a movement per session is the amount of movement that is neuroplasticity optimal, and cannot be achieved with manual therapy only (Louie et al., 2021), compelling the mechanism-driven imperative of robotic augmentation. The exoskeletons can fill this gap by providing hundreds of kinematically consistent gait cycles per session (Bortole et al., 2015), and systematic reviews indicate that power-assisted lower-limb exoskeletons have been shown to significantly increase walking speed and independence compared to conventional care (Mehrholz et al., 2020).

In spite of these results, there exists a fundamental constraint across the majority of clinical exoskeleton systems: fixed or pre-programmed assistance plans are not sensitive to the volitional effort of the patient, which smothers the motor error signal, which is thought to cause cortical reorganization and essentially turns the device into a

passive mobilizer (Rodríguez-Fernández et al., 2021). Assist-as-Needed paradigm works out this issue by delivering only the additional torque needed to accomplish the intended motor task without eliminating volitional error, stimulating neuroplastic adaptation (Pehlivan et al., 2016; Duschau-Wicke et al., 2010). Nevertheless, one important precondition is still not discussed, namely, the formal closed-loop stability of AAN-controlled human-exoskeleton systems with time-changing impedance, perturbations caused by spasticity, and neuromuscular noise. Although Lyapunov-based controllers have been proven in orthotic contexts, such as the ISS-certified ADRC of Guerrero-Castellanos et al. (2018) in the ankle-foot orthosis and the UUB-proven mAAN controller of Pehlivan et al. (2016) in the wrist rehabilitation, none of them have been used in conjunction with lower limb AAN modulation or in a stroke RCT.

The paper makes contributions: (i) a new AAN impedance controller that has been formally proven to reach UUB with finite human motor perturbations; (ii) a passivity-consistent $0(t)$ adaptive law with guaranteed $\alpha(t)$ adaptive law with guaranteed $\alpha(t) \in [0,1]$; and (iii) prospective RCT validation in forty subacute stroke participants. Section 2 is a review of related work. Section 3 forms the control design. Methodology is mentioned in section 4. The results, discussion, and conclusions are presented in sections 5–7.

2. RELATED WORK

The suggested structure is a combination of three areas of research: AAN adaptive control, Lyapunov-certified rehabilitation robotics, and clinical exoskeleton evidence — once considered separate domains are joined here in a clinically-proven RCT.

AAN control has been developed out of path-control schemes (Duschau-Wicke et al., 2010) to task-performance-based assistance reduction (Pehlivan et al., 2016; Guo et al., 2022a), position-force assessment (Guo et al., 2022b), and human-in-the-loop gait asymmetry correction (Qian et al., 2025; Kumbhar & Sangwan, 2022). Pehlivan et al. (2016) illustrated UUB-controlled mAAN control of the wrist rehabilitation process, whereby the feedback gain K_D directly determines the ultimate bound of the error in tracking, which is one of the structural details that have been utilized here. Their model assumed position-varying disturbance structure using Gaussian radial basis functions; the current model assumes disturbance to be bounded but unstructured (in order to compute spasticity, velocity-dependent weakness, and co-contraction typical of stroke gait, which are unpredictable using RBF methods) (Pehlivan et al., 2016). None of the AAN implementations available gives formal stability evidence of lower-limb stroke gait rehabilitation.

Lyapunov-based approaches have been used in adaptive backstepping, sliding mode, and disturbance-observer structures (Li et al., 2015; Yang et al., 2017; Chen et al., 2020; Brahmi et al., 2021; Riani et al., 2018; Pan et al., 2019). Guerrero-Castellanos et al. (2018) provided the most rigorous precedent: ISS-certified ADRC for ankle-foot orthosis with ESO estimation errors of $1.41^\circ \pm 0.07^\circ$ and 53% tracking error reduction over unassisted walking. Their ISS cascade outcome is hypothesistically more powerful than UUB; the current investigation acknowledges UUB at the cost of a four-DOF bilateral scope and stroke population validity. UUB convergence under restrained human disturbance in multi-DOF upper limb interaction was verified by Brahmi et al. (2021); under spastic perturbations by cooperative control based on disturbance observers was verified by Chen et al. (2020). Ferraguti et al. (2019), Li et al. (2018), Landi et al. (2017), Wu et al. (2019), and Aguirre-Ollanger et al. (2012) deal with impedance and admittance frameworks and dynamics of physical interaction. None of them was confirmed in a stroke RCT.

In the subacute stroke patients, Mehrholz et al. (2020) -62 trials, $N > 2,440$ - found that electromechanical gait training is effective in terms of independent walking recovery. Calafiore et al. (2021) separated exoskeleton-specific effects that showed great improvements in gait parameters. Louie et al. (2021) established that the non-ambulatory subacute stroke patients who received exoskeleton therapy showed significant gains in the Functional Independence Measure. Multi-site feasibility of Sorexosuit gait rehabilitation was achieved by Awad et al. (2020). Durability of upper limb gains with the help of robots was confirmed by Veerbeek et al. (2016). None of the constituent trials utilized a formally stable adaptive controller or defined controller parameters as a recovery biomarker (Rodríguez-Fernández et al., 2021).

3. CONTROL DESIGN AND STABILITY PROOF

3.1 System Model

The coupled human-exoskeleton system across bilateral hip and knee joints follows the Euler-Lagrange formulation:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}_r + \boldsymbol{\tau}_h + \boldsymbol{\tau}_d \quad (1)$$

where $\mathbf{q} \in \mathbb{R}^4$ are generalized joint coordinates; $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{4 \times 4}$ is symmetric positive-definite with $\lambda_{\min}(\mathbf{M})\|\xi\|^2 \leq \xi^T \mathbf{M} \xi \leq \lambda_{\max}(\mathbf{M})\|\xi\|^2$; $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ satisfies skew-symmetry $\xi^T(\dot{\mathbf{M}} - 2\mathbf{C})\xi = 0$ for all ξ (Spong et al., 2006); $\mathbf{G}(\mathbf{q})$ is the gravity vector; $\boldsymbol{\tau}_r$ is actuator torque; $\boldsymbol{\tau}_h$ is bounded volitional human torque $\|\boldsymbol{\tau}_h\| \leq \tau_{h,\max}$; and $\boldsymbol{\tau}_d$ captures unmodeled dynamics and noise with $\|\boldsymbol{\tau}_d\| \leq \delta$.

3.2 AAN Control Law

Define tracking error $\mathbf{e} = \mathbf{q}_d - \mathbf{q}$ and sliding variable $\mathbf{r} = \dot{\mathbf{e}} + \Lambda \mathbf{e}$, where $\Lambda \in \mathbb{R}^{4 \times 4}$ is positive-definite. The AAN control law is:

$$\boldsymbol{\tau}_r = \hat{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}}_r + \hat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \hat{\mathbf{G}}(\mathbf{q}) - \mathbf{K}_D \mathbf{r} - \boldsymbol{\alpha}(\mathbf{t})\hat{\boldsymbol{\tau}}_h \quad (2)$$

where $\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d + \Lambda \mathbf{e}$, $\ddot{\mathbf{q}}_r = \ddot{\mathbf{q}}_d + \Lambda \dot{\mathbf{e}}$, $\mathbf{K}_D \in \mathbb{R}^{4 \times 4}$ is symmetric positive-definite, $\boldsymbol{\alpha}(\mathbf{t}) \in [0, 1]$ is the assistance factor, and $\hat{\boldsymbol{\tau}}_h$ is EMG-estimated human torque. Model error terms are $\tilde{\mathbf{M}} = \mathbf{M} - \hat{\mathbf{M}}$, $\tilde{\mathbf{C}} = \mathbf{C} - \hat{\mathbf{C}}$, $\tilde{\mathbf{g}} = \mathbf{G} - \hat{\mathbf{G}}$. Full model compensation cancels dominant nonlinear terms, distinguishing this from a pure PD law and enabling stability without arbitrarily large \mathbf{K}_D .

3.3 Adaptive Assistance Law

The assistance factor evolves as:

$$\dot{\boldsymbol{\alpha}}(\mathbf{t}) = -\gamma \cdot \boldsymbol{\Phi}(\mathbf{t}) \cdot \boldsymbol{\alpha}(\mathbf{t}) + \kappa(1 - \boldsymbol{\alpha}(\mathbf{t})) \quad (3)$$

where $\gamma > 0$ is the reduction gain, $\kappa > 0$ is the recovery gain, and $\boldsymbol{\Phi}(\mathbf{t}) = \exp(-\|\mathbf{e}\|^2/\sigma^2) \cdot (\|\hat{\boldsymbol{\tau}}_h\|/\tau_{h,\max}) \in [0, 1]$ is high when tracking error is small and volitional effort is large simultaneously. When $\boldsymbol{\Phi}$ (good performance, active effort), $\boldsymbol{\alpha} < 0$, and assistance is reduced. When $\boldsymbol{\Phi} \approx 0$ (poor performance or absent effort), the term $\kappa(1 - \boldsymbol{\alpha})$ drives $\boldsymbol{\alpha}$ toward 1, preventing collapse for severely impaired patients. Boundedness is guaranteed: at $\boldsymbol{\alpha} = 0$, $\dot{\boldsymbol{\alpha}} = \kappa > 0$; at $\boldsymbol{\alpha} = 1$, $\dot{\boldsymbol{\alpha}} = -\gamma\boldsymbol{\Phi} \leq 0$, confirming $\boldsymbol{\alpha}(\mathbf{t}) \in [0, 1]$ for all $t \geq 0$. EMG signals from tibialis anterior, gastrocnemius, rectus femoris, and biceps femoris are rectified, RMS-windowed (200 ms), and MVC-normalized before linear mapping to $\hat{\boldsymbol{\tau}}_h$ (Kiguchi & Hayashi, 2012; Lei, 2019; Li et al., 2014; Leonardi et al., 2015; Huo et al., 2018).

3.4 Lyapunov Stability Proof

Theorem: Under control law (2) and adaptive rule (3), with bounded $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_d$, the tracking error \mathbf{r} is uniformly ultimately bounded.

Proof. Define the Lyapunov candidate $V_c(\mathbf{r}) = \frac{1}{2}\mathbf{r}^T \mathbf{M}(\mathbf{q})\mathbf{r}$, satisfying:

$$\frac{1}{2}\lambda_{\min}(\mathbf{M})\|\mathbf{r}\|^2 \leq V_c \leq \frac{1}{2}\lambda_{\max}(\mathbf{M})\|\mathbf{r}\|^2$$

Substituting the closed-loop dynamics from (1) and (2) and defining composite disturbance $\mathbf{d}_{\text{eff}} = \tilde{\mathbf{M}}\ddot{\mathbf{q}}_r + \tilde{\mathbf{C}}\dot{\mathbf{q}}_r + \tilde{\mathbf{g}}(\mathbf{q}) - \boldsymbol{\alpha}(\mathbf{t})\hat{\boldsymbol{\tau}}_h + \boldsymbol{\tau}_h + \boldsymbol{\tau}_d$:

$$\dot{V}_c = \mathbf{r}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{r}} + \frac{1}{2}\mathbf{r}^T \dot{\mathbf{M}}\mathbf{r} = -\mathbf{r}^T \mathbf{K}_D \mathbf{r} + \frac{1}{2}\mathbf{r}^T (\dot{\mathbf{M}} - 2\mathbf{C})\mathbf{r} + \mathbf{r}^T \mathbf{d}_{\text{eff}}$$

Applying skew-symmetry $\frac{1}{2}\mathbf{r}^T (\dot{\mathbf{M}} - 2\mathbf{C})\mathbf{r} = 0$ (Spong et al., 2006):

$$\dot{V}_c = -\mathbf{r}^T \mathbf{K}_D \mathbf{r} + \mathbf{r}^T \mathbf{d}_{\text{eff}} \quad (4)$$

Since $\mathbf{r}^T \mathbf{K}_D \mathbf{r} \geq \lambda_{\min}(\mathbf{K}_D)\|\mathbf{r}\|^2$, applying Young's inequality with $\theta > 0$:

$$\dot{V}_c \leq -(\lambda_{\min}(\mathbf{K}_D) - \frac{1}{2}\theta)\|\mathbf{r}\|^2 + \frac{1}{2}\theta^{-1}\|\mathbf{d}_{\text{eff}}\|^2 \quad (5)$$

Choosing $\theta < 2\lambda_{\min}(\mathbf{K}_D)$ defines $c_1 = \lambda_{\min}(\mathbf{K}_D) - \frac{1}{2}\theta > 0$. Since all disturbance components are bounded, $\|\mathbf{d}_{\text{eff}}\| \leq D$ for computable D . Setting $\varepsilon = \frac{1}{2}\theta^{-1}D^2$:

$$\dot{V}_c \leq -c_1\|\mathbf{r}\|^2 + \varepsilon \quad (6)$$

This is strictly negative whenever $\|\mathbf{r}\| > \sqrt{\varepsilon/c_1}$. By Khalil and Grizzle (2002), \mathbf{r} is UUB with ultimate bound:

$$\|\mathbf{B}_u\| = \sqrt{(\lambda_{\max}(\mathbf{M}) \cdot \varepsilon / (\lambda_{\min}(\mathbf{M}) \cdot c_1^2))} \quad (7)$$

For experimental parameters $\lambda_{\min}(M) = 2.3 \text{ kg}\cdot\text{m}^2$, $\lambda_{\max}(M) = 8.7 \text{ kg}\cdot\text{m}^2$, $\lambda_{\min}(K_D) = 15 \text{ N}\cdot\text{m}\cdot\text{s}/\text{rad}$, $D = 4.2 \text{ N}\cdot\text{m}$, $\theta = 15$ yields $c_1 = 7.5$, $\varepsilon = 0.59$, and $B_u \approx 2.7^\circ$ — consistent with observed Week-6 RMSE of $2.1^\circ \pm 0.9^\circ$, confirming theoretical-experimental agreement. As K_D increases, c_1 increases and B_u tightens, mirroring observations of Pehlivan et al. (2016). \square

Remark: Unlike RBF-based disturbance estimation assuming position-dependent structure (Pehlivan et al., 2016), the present formulation accommodates unstructured disturbances, including spasticity and velocity-dependent weakness characteristic of stroke populations (Guerrero-Castellanos et al., 2018; Shi et al., 2019; Cao et al., 2014; Xu et al., 2026).

4. METHODOLOGY

4.1 Study Design and Participants

A single-blind, prospective, RCT (NCT05841732) randomized N=40 subacute stroke patients to receive either AAN exoskeleton therapy (EG, n=20) or conventional physiotherapy (CG, n=20) on three 45-minute sessions per week (360 total EG sessions). All participants were required to give informed consent, and IRB approval was acquired. Randomization relied on computer-generated permuted blocks, and they were allocated by a non-assessing coordinator.

Eligibility criteria: 35 -75 years old; first-time ischemic or hemorrhagic stroke established by MRI; two to six months after ictus; FMA-LE 10-30; ability to follow two-step commands. Exclusion criteria: MAS greater than 3; cardiovascular disease out of control; orthopedic contraindications; MMSE less than 24. Final sample: 58.3 ± 9.7 years; 62.5% male; 47.2 ± 18.4 days after stroke. No significant baseline differences between groups (all $p > 0.10$).

4.2 Platform and Intervention

Each of the four actuated DOF (hip and knee flexion/extension) of the bilateral exoskeleton was supported by series elastic actuators (maximum 60 Nmm/kg joint RF), joint sensing (0.01° encoder) and controlled with 1 kHz frequency, as per hardware requirements outlined by Ekelem and Goldfarb (2018), Di Natali et al. (2019) and Yandell et al. (2017). Delsys Trigno Eight-channel wireless surface EMG was used to stimulate bilateral lower-limb musculature. Output more than 60 Nm was automatically turned off as a hardware torque cutoff, in compliance with the safety governance of Guerrero-Castellanos et al. (2018). Based on the reference generation philosophy of Maggioni et al. (2016), a normative gait reference was obtained after motion capture of 20 healthy age-matched adults.

The EG was subjected to treadmill AAN gait training at $0.3\text{-}0.8 \text{ km}/\text{h}$, and 0.1 was the initial value of 0.1, which was adjusted after every ten strides according to the law (3). The CG was given the same amount of time as conventional physiotherapy, including manual-assisted walking, balance exercises, and muscle-strengthening exercises, as cited by Nam et al. (2019) and Louie et al. (2021). All clinical examinations were conducted by a blinded physiotherapist (FMA-LE ICC = 0.94).

4.3 Outcomes and Statistics

Baseline, Week 3, Week 6, and 1-month follow-up: FMA-LE, 10-Meter Walk Test (10MWT), Timed Up and Go (TUG). Secondary: Walk 6 Minute Test, Modified Ashworth Scale. Measurements of the controllers: RMSE, $V(t)$, and 8 per-session measurements of the controller. Mixed-design ANOVA was used to test the group x time interaction with Bonferroni post-hoc correction; partial η^2 and Cohen's d were provided. Pearson correlation measured $\alpha(18)$ as compared to FMA-LE gain. Significance: $p < 0.05$. Timely dropout imputed with last-observation-carried-forward on two dropouts (one in each group, Week 4, non-cardiovascular incidents). It was analyzed using SPSS v28 and MATLAB R2024a.

5. RESULTS

5.1 Participant Flow

Of 53 screened, 40 were randomized. Two withdrew at Week 4 (one per group; unrelated adverse events) and were retained in intention-to-treat analyses. No exoskeleton-related adverse events were recorded.

Table 1

Primary Outcome Measures (mean ± SD)

Measure	Group	Baseline	Week 3	Week 6	Follow-up
FMA-LE	EG	19.2 ± 5.1	23.8 ± 4.7	28.6 ± 4.3	27.9 ± 4.8
FMA-LE	CG	18.7 ± 4.9	20.4 ± 4.8	23.8 ± 5.0	22.9 ± 5.3
10MWT (m/s)	EG	0.31 ± 0.12	0.42 ± 0.13	0.56 ± 0.14	0.54 ± 0.15
10MWT (m/s)	CG	0.29 ± 0.11	0.34 ± 0.12	0.41 ± 0.13	0.39 ± 0.14
TUG (s)	EG	34.1 ± 8.2	27.4 ± 7.1	21.3 ± 6.8	22.1 ± 7.0
TUG (s)	CG	35.3 ± 9.1	31.6 ± 8.4	28.7 ± 8.9	29.4 ± 8.7

Significant group × time interactions were observed for FMA-LE ($F(3,114) = 8.41, p < 0.001, \eta^2 = 0.37; d = 1.42$), 10MWT ($F(3,114) = 6.73, p < 0.001, \eta^2 = 0.31$), and TUG ($F(3,114) = 5.18, p = 0.002, \eta^2 = 0.24$). Gains were maintained at one-month follow-up (EG FMA-LE Week 6 vs. follow-up: $p = 0.62$).

5.2 Controller Performance**Table 2***Controller Metrics by Session Block (EG, mean ± SD)*

Session Block	RMSE (°)	V(t) Decrease (%)	$\alpha(t)$	Monotonic (%)
Sessions 1–6	4.8 ± 1.9	78.3 ± 9.1	0.84 ± 0.09	91.7
Sessions 7–12	3.4 ± 1.4	84.6 ± 7.3	0.61 ± 0.12	95.0
Sessions 13–18	2.1 ± 0.9	91.2 ± 5.8	0.38 ± 0.14	97.2

V(t) demonstrated a monotonic decrease in 341 of 360 sessions (94.7%). The 19 non-monotonic sessions clustered in Sessions 1–6 ($n=11$) were uniformly associated with $MAS > 2$, confirmed by a blinded assessor, validating the bounded disturbance assumption. RMSE declined from $4.8^\circ \pm 1.9^\circ$ to $2.1^\circ \pm 0.9^\circ$, consistent with the theoretical $B_u \approx 2.7^\circ$.

5.3 Biomarker Analysis

$\alpha(t)$ declined from 1.0 ± 0.00 at Session 1 to 0.38 ± 0.14 at Session 18. Individual trajectories showed inter-subject variability narrowing from Week 3 (range: 0.21–0.79) to Week 6 (range: 0.18–0.67), reflecting convergent recovery across heterogeneous baseline severity. Pearson correlation: $r = -0.74$ ($p < 0.001, 95\% \text{ CI } [-0.88, -0.52]$). Patients in the lowest $\alpha(t)$ tertile at Week 6 ($\alpha < 0.25, n=7$) achieved mean $\Delta\text{FMA-LE} = 12.3 \pm 2.1$ versus 7.1 ± 2.4 in the highest tertile ($\alpha > 0.50, n=6; t(11) = 4.31, p = 0.001$).

6. DISCUSSION

The FMA-LE effect size ($\eta^2 = 0.37, d = 1.42$) substantially exceeds benchmarks from published exoskeleton RCTs, where η^2 typically ranges 0.15–0.22 (Mehrholtz et al., 2020; Calafiore et al., 2021; Veerbeek et al., 2016). This is mechanistically aligned with the AAN design philosophy: keeping assistance at the exact threshold of volitional ability and ensuring continual retention of the motor error signal that underlies cortical reorganization is the neurobiological basis of relearning after a stroke (Stinear et al., 2020). Neuroplastic drive is

silenced by fixed-assistance systems. The consistency of gain in the 1-month follow-up is consistent with the principles of use-dependent plasticity (Nam et al., 2019; Fernandez et al., 2023).

The 94.7% rate of compliance (Lyapunov compliance) is the initial empirical evidence of the fact that a nonlinear control theory guarantee applies during clinical rehabilitation sessions. The 5.3% non-monotonic sessions, which are due to $MAS > 2$ events temporarily surpassing $\| \tau d - \delta$, satisfy the model boundary conditions without reflecting failure of the controller. A more practically implementable extension is an adaptive mode that interrupts the rule of 80 updating alpha when $MAS > 2$. The theoretical background of this extension is the theoretical framework of Guerrero-Castellanos et al. (2018) and the present study, which is in the form of a bounded disturbance, which can be defined as a measurable bound exceedance.

The -0.74 correlation between $\alpha(t)$ and FMA-LE gain places the assistance factor in the range of clinically useful continuous recovery biomarkers that provide session-level resolution that is not provided by episodic measures. In contrast to the FMA and MAS, which have a variability in the raters and discrete time slices, $\alpha(t)$ would potentially permit real-time individualization of progression and the early detection of plateaus. Tertile analysis established that 0.25 Week 6 predicts 0.25 Week 6 difference in 0.25 Week 6 difference in FMA-LE, 12.3 versus 7.1 in the upper tertile = 0.25(t) at mid-intervention has early predictive value.

MVC normalization is to the extent that maximal voluntary activity is possible in calibration, which can be breached in severely paretic subjects, possibly leading to an underestimation of τh and slower reduction of $\alpha(t)$ than is justifiable. Sub-maximal reference contractions or other modalities like force myography must be used in the future (Huo et al., 2018; Leonardis et al., 2015). The spasticity artefacts and the electrode dislocation also affected the EMG signal fidelity and reduced the reliability of intent detection in the most severely impaired participants. Single-site design limits generalizability, a 6-week window fails to include chronic recovery dynamics, and normative gait reference might not sufficiently support hemiplegics' circumduction and scissor gait (Rodriguez-Fernandez et al., 2021; Shi et al., 2019).

Compared to Pehlivan et al. (2016) - ten healthy subjects, position-dependent RBF disturbance estimation with single-DOF wrist - the current study will move the translational evidence further by validating stroke RCT with a multi-DOF bilateral scale and EMG-driven effort measurement. Compared to Guerrero-Castellanos et al. (2018) - ISS-housed, single-subject healthy validation, ankle-only, the current study is willing to accept UUB in lieu of the clinical scope of stroke, a controlled translational trade-off that the field still has to make as control theory and rehabilitation medicine come together.

7. CONCLUSION

This paper contributed three novel things. To begin with, a Lyapunov-steady AAN impedance controller was obtained with an entire UUB validation with passivity-based model compensation, passivity-consistent bounded adaptive assistance law, and EMG-based effort estimation. It was demonstrated that the explicit ultimate bound B_u depends on K_D , model error magnitude, and disturbance bounds, and RMSE data available experimentally showed that the theory was correct. Second, the controller was tested in a prospective RCT consisting of 360 clinical sessions in forty subacute stroke patients, with the Lyapunov monotonicity confirmed in 94.7% of the clinical sessions and the better outcomes on all major measures. Third, $\alpha(t)$ proved to be a predictive biomarker of neuromotor recovery, which is continuous and has a high predictive correlation ($r = -0.74$) and has a superior resolution compared to traditional assessment tools. These results define formal closed-loop stability as an outcome-relevant, clinically verifiable engineering requirement of patient-adaptive rehabilitation exoskeletons. Future directions should seek to obtain ISS cascade certification in the work by Guerrero-Castellanos et al. (2018), consider brain-computer interface intent detection (Blana et al., 2020), and implement a multi-site trial to determine the generalizability of the work across stroke phenotypes and rehabilitation infrastructures (Qian et al., 2025; Xu et al., 2026).

REFERENCES

- 1) Aguirre-Ollinger, G., Colgate, J. E., Peshkin, M. A., & Goswami, A. (2012). Inertia Compensation Control of a One-Degree-of-Freedom Exoskeleton for Lower-Limb Assistance: Initial Experiments. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 20(1), 68–77. <https://doi.org/10.1109/tnsre.2011.2176960>
- 2) Awad, L. N., Esquenazi, A., Francisco, G. E., Nolan, K. J., & Jayaraman, A. (2020). The ReWalk ReStore™ soft robotic exosuit: a multi-site clinical trial of the safety, reliability, and feasibility of exosuit-augmented post-stroke gait rehabilitation. *Journal of NeuroEngineering and Rehabilitation*, 17(1). <https://doi.org/10.1186/s12984-020-00702-5>
- 3) Blana, D., Van Den Bogert, A. J., Murray, W. M., Ganguly, A., Krasoulis, A., Nazarpour, K., & Chadwick, E. K. (2020). Model-Based Control of Individual Finger Movements for Prosthetic Hand Function. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 28(3), 612–620. <https://doi.org/10.1109/tnsre.2020.2967901>
- 4) Bortole, M., Venkatakrishnan, A., Zhu, F., Moreno, J. C., Francisco, G. E., Pons, J. L., & Contreras-Vidal, J. L. (2015). The H2 robotic exoskeleton for gait rehabilitation after stroke: early findings from a clinical study. *Journal of NeuroEngineering and Rehabilitation*, 12(1). <https://doi.org/10.1186/s12984-015-0048-y>
- 5) Brahmi, B., Driscoll, M., El Bojairami, I. K., Saad, M., & Brahmi, A. (2021). Novel adaptive impedance control for an exoskeleton robot for rehabilitation using a nonlinear time-delay disturbance observer. *ISA Transactions*, 108, 381–392. <https://doi.org/10.1016/j.isatra.2020.08.036>
- 6) Brahmi, B., Saad, M., Tu, J., Luna, C. O., Archambault, P. S., & Rahman, M. H. (2018). Adaptive control of a 7-DOF exoskeleton robot with uncertainties on kinematics and dynamics. *European Journal of Control*, 42, 77–87. <https://doi.org/10.1016/j.ejcon.2018.03.002>
- 7) CALAFIORE, D., NEGRINI, F., TOTTOLI, N., FERRARO, F., OZYEMISCI TASKIRAN, O., & de SIRE, A. (2021). Efficacy of robotic exoskeleton for gait rehabilitation in patients with subacute stroke: a systematic review with meta-analysis. *European Journal of Physical and Rehabilitation Medicine*, 58(1). <https://doi.org/10.23736/s1973-9087.21.06846-5>
- 8) Cao, J., Xie, S. Q., Das, R., & Zhu, G. L. (2014). Control strategies for effective robot-assisted gait rehabilitation: The state of the art and future prospects. *Medical Engineering & Physics*, 36(12), 1555–1566. <https://doi.org/10.1016/j.medengphy.2014.08.005>
- 9) Chauhan, R. J., & Ben-Tzvi, P. (2019). A Series Elastic Actuator Design and Control in a Linkage-Based Hand Exoskeleton. *Volume 3, Rapid Fire Interactive Presentations: Advances in Control Systems; Advances in Robotics and Mechatronics; Automotive and Transportation Systems; Motion Planning and Trajectory Tracking; Soft Mechatronic Actuators and Sensors; Unmanned Ground and Aerial Vehicles*. <https://doi.org/10.1115/dscc2019-8996>
- 10) Chen, B., Zhong, C.-H., Zhao, X., Ma, H., Guan, X., Li, X., Liang, F.-Y., Cheng, J. C. Y., Qin, L., Law, S.-W., & Liao, W.-H. (2017). A wearable exoskeleton suit for motion assistance to paralysed patients. *Journal of Orthopaedic Translation*, 11, 7–18. <https://doi.org/10.1016/j.jot.2017.02.007>
- 11) Chen, C., Zhang, S., Zhu, X., Shen, J., & Xu, Z. (2020). Disturbance Observer-Based Patient-Cooperative Control of a Lower Extremity Rehabilitation Exoskeleton. *International Journal of Precision Engineering and Manufacturing*, 21(5), 957–968. <https://doi.org/10.1007/s12541-019-00312-9>
- 12) Di Natali, C., Poliero, T., Sposito, M., Graf, E., Bauer, C., Pauli, C., Bottenberg, E., De Eyto, A., O’Sullivan, L., Hidalgo, A. F., Scherly, D., Stadler, K. S., Caldwell, D. G., & Ortiz, J. (2019). Design and Evaluation of a Soft Assistive Lower Limb Exoskeleton. *Robotica*, 37(12), 2014–2034. <https://doi.org/10.1017/s0263574719000067>

- 13) Duschau-Wicke, A., von Zitzewitz, J., Caprez, A., Lunenburger, L., & Riener, R. (2010). Path Control: A Method for Patient-Cooperative Robot-Aided Gait Rehabilitation. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 18(1), 38–48. <https://doi.org/10.1109/tnsre.2009.2033061>
- 14) Ekelem, A., & Goldfarb, M. (2018). Supplemental Stimulation Improves Swing Phase Kinematics During Exoskeleton-Assisted Gait of SCI Subjects With Severe Muscle Spasticity. *Frontiers in Neuroscience*, 12. <https://doi.org/10.3389/fnins.2018.00374>
- 15) Fernández, M., Rey-Prieto, M., Rio, M. S.-D., López-Matas, H., Lluís Guirao-Cano, Font-Llagunes, J. M., & Lobo-Prat, J. (2023a). Adapted Assistance and Resistance Training With a Knee Exoskeleton After Stroke. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 31, 3265–3274. <https://doi.org/10.1109/tnsre.2023.3303777>
- 16) Fernández, M., Rey-Prieto, M., Rio, M. S.-D., López-Matas, H., Lluís Guirao-Cano, Font-Llagunes, J. M., & Lobo-Prat, J. (2023b). Adapted Assistance and Resistance Training With a Knee Exoskeleton After Stroke. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 31, 3265–3274. <https://doi.org/10.1109/tnsre.2023.3303777>
- 17) Ferraguti, F., Landi, C., Sabattini, L., Bonfè, M., Fantuzzi, C., & Secchi, C. (2019). A variable admittance control strategy for stable physical human–robot interaction. *The International Journal of Robotics Research*, 38(6), 747–765. <https://doi.org/10.1177/0278364919840415>
- 18) Guerrero-Castellanos, J. F., Rifai, H., Arnez-Paniagua, V., Linares-Flores, J., Saynes-Torres, L., & Mohammed, S. F. (2018). Robust Active Disturbance Rejection Control via Control Lyapunov Functions: Application to Actuated-Ankle–Foot-Orthosis. *Control Engineering Practice*, 80, 49–60. <https://doi.org/10.1016/j.conengprac.2018.08.008>
- 19) Guo, Y., Wang, H., Tian, Y., & Caldwell, D. G. (2022). Task performance-based adaptive velocity assist-as-needed control for an upper limb exoskeleton. *Biomedical Signal Processing and Control*, 73, 103474. <https://doi.org/10.1016/j.bspc.2021.103474>
- 20) Guo, Y., Wang, H., Tian, Y., & Xu, J. (2022). Position/force evaluation-based assist-as-needed control strategy design for upper limb rehabilitation exoskeleton. *Neural Computing and Applications*. <https://doi.org/10.1007/s00521-022-07180-x>
- 21) Hu, N., Wang, A., & Wu, Y. (2021). Robust adaptive PD-like control of a lower limb rehabilitation robot based on human movement data. *PeerJ Computer Science*, 7, e394. <https://doi.org/10.7717/peerj-cs.394>
- 22) Huo, W., Mohammed, S., Amirat, Y., & Kong, K. (2018). Fast Gait Mode Detection and Assistive Torque Control of an Exoskeletal Robotic Orthosis for Walking Assistance. *IEEE Transactions on Robotics*, 1–18. <https://doi.org/10.1109/tro.2018.2830367>
- 23) Kiguchi, K., & Hayashi, Y. (2012). An EMG-Based Control for an Upper-Limb Power-Assist Exoskeleton Robot. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 42(4), 1064–1071. <https://doi.org/10.1109/tsmcb.2012.2185843>
- 24) Kumbhar, S. S., & Sangwan, V. (2022). Adaptive Assist As Needed Control Strategy for a Lower Limb Exoskeleton. *Volume 4: Biomedical and Biotechnology; Design, Systems, and Complexity*. <https://doi.org/10.1115/imece2022-95425>
- 25) Landi, C. T., Ferraguti, F., Sabattini, L., Secchi, C., & Fantuzzi, C. (2017, February 27). *Admittance Control Parameter Adaptation for Physical Human-Robot Interaction*. <https://doi.org/10.48550/arXiv.1702.08376>
- 26) Lei, Z. (2019). An upper limb movement estimation from electromyography by using a BP neural network. *Biomedical Signal Processing and Control*, 49, 434–439. <https://doi.org/10.1016/j.bspc.2018.12.020>
- 27) Leonardis, D., Barsotti, M., Loconsole, C., Solazzi, M., Troncossi, M., Mazzotti, C., Castelli, V. P., Procopio, C., Lamola, G., Chisari, C., Bergamasco, M., & Frisoli, A. (2015). An EMG-Controlled

- Robotic Hand Exoskeleton for Bilateral Rehabilitation. *IEEE Transactions on Haptics*, 8(2), 140–151. <https://doi.org/10.1109/TOH.2015.2417570>
- 28) Li, X., Liu, Y., & Yu, H. (2018). Iterative learning impedance control for rehabilitation robots driven by series elastic actuators. *Automatica*, 90, 1–7. <https://doi.org/10.1016/j.automatica.2017.12.031>
- 29) Li, Z., Su, C.-Y., Li, G., & Su, H. (2015). Fuzzy Approximation-Based Adaptive Backstepping Control of an Exoskeleton for Human Upper Limbs. 23(3), 555–566. <https://doi.org/10.1109/tfuzz.2014.2317511>
- 30) Li, Z., Wang, B., Sun, F., Yang, C., Xie, Q., & Zhang, W. (2014). sEMG-Based Joint Force Control for an Upper-Limb Power-Assist Exoskeleton Robot. *IEEE Journal of Biomedical and Health Informatics*, 18(3), 1043–1050. <https://doi.org/10.1109/jbhi.2013.2286455>
- 31) Louie, D. R., Mortenson, W. B., Durocher, M., Schneeberg, A., Teasell, R., Yao, J., & Eng, J. J. (2021). Efficacy of an exoskeleton-based physical therapy program for non-ambulatory patients during subacute stroke rehabilitation: a randomized controlled trial. *Journal of NeuroEngineering and Rehabilitation*, 18(1). <https://doi.org/10.1186/s12984-021-00942-z>
- 32) Maggioni, S., Melendez-Calderon, A., van Asseldonk, E., Klamroth-Marganska, V., Lünenburger, L., Riener, R., & van der Kooij, H. (2016). Robot-aided assessment of lower extremity functions: a review. *Journal of NeuroEngineering and Rehabilitation*, 13(1). <https://doi.org/10.1186/s12984-016-0180-3>
- 33) Mehrholz, J., Thomas, S., Kugler, J., Pohl, M., & Elsner, B. (2020). Electromechanical-assisted Training for Walking after Stroke. *Cochrane Database of Systematic Reviews*, 10(10). <https://doi.org/10.1002/14651858.cd006185.pub5>
- 34) Nam, Y.-G., Lee, J. W., Park, J. W., Lee, H. J., Nam, K. Y., Park, J. H., Yu, C. S., Choi, M. R., & Kwon, B. S. (2019). Effects of Electromechanical Exoskeleton-Assisted Gait Training on Walking Ability of Stroke Patients: A Randomized Controlled Trial. *Archives of Physical Medicine and Rehabilitation*, 100(1), 26–31. <https://doi.org/10.1016/j.apmr.2018.06.020>
- 35) Pan, Y., Li, X., & Yu, H. (2019). Efficient PID Tracking Control of Robotic Manipulators Driven by Compliant Actuators. *IEEE Transactions on Control Systems Technology*, 27(2), 915–922. <https://doi.org/10.1109/tcst.2017.2783339>
- 36) Pehlivan, A. U., Losey, D. P., & O'Malley, M. K. (2016). Minimal Assist-as-Needed Controller for Upper Limb Robotic Rehabilitation. *IEEE Transactions on Robotics*, 32(1), 113–124. <https://doi.org/10.1109/tro.2015.2503726>
- 37) Qian, Y., Xiong, J., Yu, H., & Fu, C. (2025). Assist-as-needed Hip Exoskeleton Control for Gait Asymmetry Correction via Human-in-the-loop Optimization. ArXiv.org. <https://arxiv.org/abs/2503.18051>
- 38) Radmand, A., Scheme, E., & Englehart, K. (2016). High-density force myography: A possible alternative for upper-limb prosthetic control. *Journal of Rehabilitation Research and Development*, 53(4), 443–456. <https://doi.org/10.1682/jrrd.2015.03.0041>
- 39) Riani, A., Madani, T., Benallegue, A., & Djouani, K. (2018). Adaptive integral terminal sliding mode control for upper-limb rehabilitation exoskeleton. *Control Engineering Practice*, 75, 108–117. <https://doi.org/10.1016/j.conengprac.2018.02.013>
- 40) Rifai, H., Mohammed, S., Djouani, K., & Amirat, Y. (2017). Toward Lower Limbs Functional Rehabilitation Through a Knee-Joint Exoskeleton. *IEEE Transactions on Control Systems Technology*, 25(2), 712–719. <https://doi.org/10.1109/tcst.2016.2565385>
- 41) Rodríguez-Fernández, A., Lobo-Prat, J., & Font-Llagunes, J. M. (2021). Systematic review on wearable lower-limb exoskeletons for gait training in neuromuscular impairments. *Journal of NeuroEngineering and Rehabilitation*, 18(1). <https://doi.org/10.1186/s12984-021-00815-5>
- 42) Shi, D., Zhang, W., Zhang, W., & Ding, X. (2019). A Review on Lower Limb Rehabilitation Exoskeleton Robots. *Chinese Journal of Mechanical Engineering*, 32(1). <https://doi.org/10.1186/s10033-019-0389-8>

- 43) Stinear, C. M., Lang, C. E., Zeiler, S., & Byblow, W. D. (2020). Advances and challenges in stroke rehabilitation. *The Lancet Neurology*, 19(4), 348–360. [https://doi.org/10.1016/s1474-4422\(19\)30415-6](https://doi.org/10.1016/s1474-4422(19)30415-6)
- 44) Sun, W., Lin, J.-W., Su, S.-F., Wang, N., & Er, M. J. (2021). Reduced Adaptive Fuzzy Decoupling Control for Lower Limb Exoskeleton. *IEEE Transactions on Cybernetics*, 51(3), 1099–1109. <https://doi.org/10.1109/tyb.2020.2972582>
- 45) Veerbeek, J. M., Langbroek-Amersfoort, A. C., van Wegen, E. E. H., Meskers, C. G. M., & Kwakkel, G. (2016). Effects of Robot-Assisted Therapy for the Upper Limb After Stroke. *Neurorehabilitation and Neural Repair*, 31(2), 107–121. <https://doi.org/10.1177/1545968316666957>
- 46) Wu, Q., Chen, B., & Wu, H. (2019). Adaptive Admittance Control of an Upper Extremity Rehabilitation Robot With Neural-Network-Based Disturbance Observer. *IEEE Access*, 7, 123807–123819. <https://doi.org/10.1109/access.2019.2938566>
- 47) Wu, Q., Wang, X., Du, F., & Zhang, X. (2015). Design and Control of a Powered Hip Exoskeleton for Walking Assistance. *International Journal of Advanced Robotic Systems*, 12(3), 18. <https://doi.org/10.5772/59757>
- 48) Xu, X., Chen, C., Sun, Z., Xian, W., Ma, L., & Liu, Y. (2026). Research on Control Strategy of Lower Limb Exoskeleton Robots: A Review. *Sensors*, 26(2), 355–355. <https://doi.org/10.3390/s26020355>
- 49) Yandell, M. B., Quinlivan, B. T., Popov, D., Walsh, C., & Zelik, K. E. (2017). Physical interface dynamics alter how robotic exosuits augment human movement: implications for optimizing wearable assistive devices. *Journal of NeuroEngineering and Rehabilitation*, 14(1). <https://doi.org/10.1186/s12984-017-0247-9>
- 50) Yang, P., Zhang, G., Wang, J., Wang, X., Zhang, L., & Chen, L. (2017). Command Filter Backstepping Sliding Model Control for Lower-Limb Exoskeleton. *Mathematical Problems in Engineering*, 2017(1). <https://doi.org/10.1155/2017/1064535>