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EFFECT OF DAMPING COEFFICIENT ON TRANSVERSE DISPLACEMENT AND ROTATION UNDER MOVING LOAD OF ELASTICALLY SUPPORTED PRESTRESSED SHEAR BEAM

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ABSTRACT
This study investigates the effect of the damping coefficient on the transverse displacement and rotation of a prestressed shear beam resting on an elastic foundation and subjected to a moving load at a constant velocity. The governing equations are coupled second-order partial differential equations. To facilitate the analysis, the finite Fourier series method was utilized, converting these equations into a set of coupled second-order ordinary differential equations. The resulting equations that characterize the motion of the beam-load system were then solved using Laplace transformation alongside convolution theory to derive the solutions. Analyses were performed to assess the effect of the damping coefficient on both the transverse displacement and rotation of prestressed shear beams of different lengths when subjected the moving load at different velocities respectively. Furthermore, the research investigates the effect of the damping coefficient on the critical velocities of the vibrating system. The results indicate that the transverse displacement and rotation of the beam significantly decrease as the damping coefficient increases. Additionally, it was observed that an increase in the damping coefficient corresponds to increase in the critical velocity, suggesting a more stable dynamic system. Practically, this connotes the significance of the damping coefficient in enhancing the dynamic stability of the beam under the influence of a moving load.

Keywords:
Damping Coefficient, Transverse Displacement, Rotation, Moving Load, Prestressed, Shear Beam

INTRODUCTION

The analysis of vibration induced by moving loads in various applications of vibration and structural dynamics has been the focus of extensive research over the past century. This study continues to be of significant interest due to the widespread applications of structure-moving load interactions in fields such as Civil, Mechanical, Aerospace, and Structural Engineering, among others. In structural dynamics, moving loads are defined as loads that change position over time on a structure. These dynamic loads can induce vibrations and alter stress within the structure, necessitating thorough analysis and design [1-3]. Real-life examples of moving-load-induced vibrations include those experienced in interactions between vehicles and bridges, human interactions with footbridges, railway track dynamics, cranes operating on rails, and high-speed machining processes. Therefore, a comprehensive understanding of structure-moving load interactions is essential for the design of safe and effective structures in practical scenarios. A significant amount of literature has been devoted to exploring moving load problems, with numerous researchers investigating how the complex interactions between beams or beam-like structural elements and the loads traversing them affect the dynamic behavior of these structures [4-7].

Furthermore, beams that are supported by elastic foundations, which are often represented as beams on springs, are commonly analyzed in various structures such as buildings, bridges, roads, pavements, railway tracks, and many other related constructions. Elastic foundations illustrate the soil's capacity to deform under load and distribute stresses, thereby preventing excessive settlement or structural damage. They play a vital role in engineering by enabling accurate predictions of structural behavior under load, particularly in scenarios where

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structures interact with soil or other supporting materials. These models simplify the intricate soil-structure interaction, facilitating efficient analysis of deflections, stresses, and stability across various engineering applications. The study of vibrations in beam-like structures resting on elastic foundations due to moving loads is of considerable technological importance and it has been extensively studied. Notably, Ajijola [8] studied the dynamic response to moving load of prestressed damped shear beam resting on bi-parametric elastic foundation. Ogunbamike [9] considered the dynamic behavior of a simply supported Timoshenko beam on a Winkler foundation subjected to a moving uniformly distributed load. Additionally, Clastornik et al. [10] conducted research on the dynamic analysis of elastic beams supported by a variable Winkler elastic foundation. Similarly, Ogunbamike and Oni [11] investigated the dynamic characteristics of a non-prismatic Rayleigh beam with general boundary conditions, supported by a Vlasov elastic foundation and subjected to partially distributed moving masses with varying velocities. They employed the Generalized Galerkin method to derive closed-form solutions for this category of dynamic problems. Also, Rajib et al. [12] explored the dynamic response of a beam under both moving loads and moving masses, supported by a Pasternak foundation.

Additionally, engineers frequently introduce artificial stresses into structures or systems to enhance their performance. This technique, known as pre-stressing, offers significant advantages, such as minimizing tensile stresses and reducing the risk of flexural cracking or bending during operational conditions. This method is vital in structural engineering and mechanics, as it increases the load-bearing capacity and improves the durability, strength, and stability of structures and machinery. As a result, extensive research has focused on the vibrations of pre-stressed beams under moving loads. For instance, Ajijola [13] considered Axial force influence on transverse displacement and rotation under moving load of elastically supported damped shear beam. Jimoh, Oni and Ajijola [14] examined how variable axial forces affect the deflection of thick beams under distributed moving loads, calculating the transverse displacement for various time intervals and analyzing the results. Similarly, Jimoh, Ogunbamike, and Ajijola [15] explored the dynamic response of non-uniformly prestressed thick beams under distributed moving loads at different velocities. They employed a technique based on Galerkin's method, utilizing the series representation of the Heaviside function to transform the equations, which were then simplified using Strubles asymptotic method and solved through Laplace transformation techniques combined with convolution theory. Their findings indicated that the moving distributed force does not serve as an upper bound for accurately solving the moving distributed mass problem. Furthermore, they discovered that increasing certain structural parameters leads to a reduction in the response amplitudes of non-uniformly prestressed thick beams under moving distributed loads. Ogunbamike [16] also investigated the dynamic response of a Timoshenko beam supported by an elastic foundation and subjected to a harmonic moving load. The technique of modal analysis (MA) was utilized to derive a closed-form solution for this category of dynamical systems. A comprehensive examination of the influence of axial force and foundation parameters on the dynamic behavior of the beams was conducted and thoroughly documented.

In most of the studies in the literature examined, the influence of damping on the vibrations of dynamic systems is scanty. An undamped system is assumed to oscillate freely with a constant amplitude indefinitely. However, in reality, this is not the case; every system that has mass and elasticity can oscillate, which leads to energy dissipation from the system. Thus, the energy dissipation from a vibrating or oscillating system, which effectively reduces or halts its motion, is known as damping. Damping holds significant importance in structural and construction engineering as it safeguards systems from excessive vibrations and potential damage, facilitating smooth and efficient operation. Notable contributors in this area include Crandall [17] who examined the role of damping in specific areas where minimal damping plays a crucial role in influencing a system's dynamic behavior. Mousa and Reza [18] proposed an innovative approach for the free vibrational synthesis of a cracked cantilever beam with a breaking crack, considering the impact of distributed structural damping. Robin and Rana [19] investigated the vibrations of isotropic and orthotropic damped plates with varying thicknesses resting on a foundation. Similarly, Ogunbamike [20] analyzed the effects of damping on the transverse motions of axially loaded beams subjected to distributed moving loads. This study focuses on the dynamic analysis of a clamped-clamped Rayleigh beam under moving distributed loads, employing a solution technique based on the generalized finite integral transform and a modification of Struble's asymptotic technique. Additionally, Famuagun [21] examined the influence of damping

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coefficients on the dynamic response of Rayleigh beams supported by a bi-parametric elastic foundation when subjected to moving distributed masses.

It is pertinent to remark that, despite wide-ranging researches focused on the dynamic behavior of beams subjected to moving loads, studies on damped shear beams have been exclusively circumscribed in the existing literature. Consequently, considering the great impact of damping on the dynamic behavior of beams and other similar structural elements, this present study therefore critically examines the effect of damping coefficient on the transverse displacement, rotation and critical velocity of a simply supported prestressed shear beam when traversed by a moving load traveling at a constant velocity.

OBJECTIVES

The specific objectives of this study are to;

(i) obtain the analytical solution of the governing coupled second order partial differential equations of the of a simply supported prestressed shear beam resting on bi-parametric elastic foundation when it is subjected to a moving load traveling at a constant velocity and

(ii) classify the effect of damping coefficient on the transverse displacement, rotation and critical velocity of the vibrating system.

METHODOLOGY

PROBLEM STATEMENT

The theory of shear beams is a vital aspect of structural engineering, concentrating on beams where shear deformation is significant. The shear beam model is typically defined by a pair of coupled partial differential equations that involve two dependent variables: the transverse displacement of the cross-section relative to the neutral axis and rotation of the cross section measured about the neutral axis. The equations governing the transverse displacement and rotation of shear beam under the action of moving load are based on the following assumptions [1]:

(i) The material is linearly elastic and the beam is homogeneous at any cross-section (prismatic)
(ii) The x - y plane is the principal plane.
(iii) There is an axis of the beam that undergoes no extension or contraction. The x-axis is located along this neutral axis.
(iv) Plane section remains plain after bending but is no longer normal to the longitudinal axis.
(v) The effect of shear deformation is considered.

MATHEMATICAL MODEL

The governing equations of motion describing the transverse displacement $V(x,t)$ and rotation $\phi(x,t)$ of a shear beam when subjected to a moving load traveling at a constant velocity are formulated as coupled second order partial differential equations given by [13]:

$$M\frac{\partial^2 V(x,t)}{\partial t^2} + \frac{\partial}{\partial x}\left[K^* G^* A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] = P^*(x,t) \quad (1)$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \phi(x,t)}{\partial x}\right) - K^* G^* A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right) = 0 \quad (2)$$

where M is the mass per unit length of the beam, K^* is the shear correction factor, G^* is the shear parameter of the beam, A is the cross-sectional area of the beam, E is the Young modulus of elasticity of the beam material, I is the

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moment of inertia, EI is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate and $P^*(x,t)$ is the moving load acting on the beam per unit length.

The relationship between the foundation reaction $F^*(x,t)$ and transverse displacement $V(x,t)$ is given by

$$F^*(x,t) = KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} \tag{3}$$

where K and G are two parameters of the foundation model. Specifically, K is the Foundation Stiffness and G is the Shear Modulus.
In this study, it is assumed that the load function $P^*(x,t)$ is given in the form

$$P^*(x,t) = P_0\delta(x - ct). \tag{4}$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property.

$$\int_{k_0}^{k_1} \partial(x - ct)f(x)dx = \{\, 0,\ \ for\ \ ct < k_0 < k_1,\quad f(ct),\ \ for\ \ k_0 < ct < k_1,\quad 1,\ \ for\ \ k_0 < k_1 < ct. \tag{5}$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x = 0$ and $x = L$. Thus, boundary conditions are given as

$$V(0,t) = V(L,t) = 0,\qquad \frac{\partial V(0,t)}{\partial x} = \frac{\partial V(L,t)}{\partial x} = 0\quad \phi(0,t) = \phi(L,t) = 0,$$
$$\frac{\partial \phi(0,t)}{\partial x} = \frac{\partial \phi(L,t)}{\partial x} = 0 \tag{6}$$

and the initial conditions are given as

$$V(0,x) = 0 = \frac{\partial V(x,0)}{\partial t},\qquad \phi(0,x) = \ 0 = \frac{\partial \phi(x,0)}{\partial t} \tag{7}$$

Now, introducing damping and axially prestressed parameters and in view of (3) and (4), after some simplifications and re-arrangements, equations (1) and (2) become

$$\frac{\partial}{\partial x}\left[K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] + M\frac{\partial^2 V(x,t)}{\partial t^2} - N_0\frac{\partial^2 V(x,t)}{\partial x^2} - C_0\frac{\partial V(x,t)}{\partial t} + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} = P_0\delta(x - ct) \tag{8}$$

and

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \phi(x,t)}{\partial x}\right) - K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right) = 0 \tag{9}$$

where N_0 is the axial force and C_0 is the coefficient of damping per unit length of the beam.

Hence, $(8)\ and (9)$ are the second order partial differential equations governing the flexural motion of an elastically supported prestressed shear beam when subjected to a moving load traveling at a constant velocity.

SOLUTION PROCEDURES

The shear beam examined in this study is both finite and uniform. In order to obtain the analytical solution for the initial boundary value problem in equations (8) and (9), we employ the finite Fourier transformation method alongside the Laplace Transform. Subsequently, we present the following definitions and theorem [13].

Definition 1: The finite Fourier sine transform $w(n,t)$ of a function $W(x,t)$ is defined as

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$$w(n,t) = \int_0^l W(x,t) \sin \frac{n\pi x}{l}\, dx \tag{10}$$

and the inverse transform is

$$W(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} w(n,t) \sin \frac{n\pi x}{l}\, dx. \tag{11}$$

Definition 2: The finite Fourier cosine transform $w_0(n,t)$ of a function $W_0(x,t)$ is defined as

$$w_0(n,t) = \int_0^l W_0(x,t) \cos \frac{n\pi x}{l}\, dx \tag{12}$$

and the inverse transform is

$$W_0(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} w_0(n,t) \cos \frac{n\pi x}{l}\, dx. \tag{13}$$

Definition 3: The Laplace transform $F(s)$ of a function $f(t)$ is defined as

$$L\big(f(t)\big) = F(s) = \int_0^{\infty} f(t)\, e^{-st} dt\,. \tag{14}$$

Theorem 1: The convolution theorem states that

$$L^{-1}\{F(s)G(s)\} = F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{15}$$

where $F(s)$ and $G(s)$ are the Laplace transforms of $f(t)$ and $g(t)$ respectively.

Consequently, applying (10) and (12) to the governing equations (8) and (9) respectively, along with the property of the Dirac delta function as stated in (5), we obtain

$$\frac{\partial^2 V(n,t)}{\partial t^2} + \mu_1 \frac{\partial V(n,t)}{\partial t} + \mu_2 V(n,t) - \mu_3 \frac{\partial \phi(n,t)}{\partial x} = Q_1 \sin \theta_n t \tag{16}$$

and

$$\phi(n,t) = \mu_0 V(n,t) \tag{17}$$

where

$$\mu_1 = -\frac{C_0}{M},\quad \mu_2 = \left(\frac{n\pi}{ML}\right)^2 (N_0 + G) - \frac{K}{M},\quad \mu_3 = \left(\frac{n\pi}{ML}\right) K^* G^* A,$$

$$Q_1 = \frac{P_0}{M},\quad \theta_n = \frac{n\pi c}{L},\quad \mu_0 = \frac{\frac{n\pi}{L} K^* G^* A}{EI\left(\frac{n\pi}{L}\right)^2 + K^* G^* A}$$

Now putting (17) into (16), we have

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$$\frac{\partial^2 V(n,t)}{\partial t^2} + \mu_1 \frac{\partial V(n,t)}{\partial t} + \mu_2 V(n,t) - \mu_3 \frac{\partial}{\partial x}\big(\mu_0 V(n,t)\big) = Q_1 \sin\theta_n t \tag{18}$$

The term involving the derivative with respect to x in (18) vanishes as $V(n,t)$ is a function of t alone. Following some simplifications and re-arrangements, we obtain

$$V_{tt}(n,t) + \mu_1 V_t(n,t) + \mu_4 V(n,t) = Q_1 \sin\theta_n t \tag{19}$$

where

$$\mu_4 = \mu_2 - \mu_0\mu_3$$

Now, subjecting (19) to Laplace transformation (14), namely

$$\mathcal{L}\big(f(t)\big) = \mathrm{F(s)} = \int_0^\infty f(t)\, e^{-st} dt \tag{20}$$

where s is the Laplace parameter. In view of (20), (19) becomes

$$s^2\tilde{V}(n,s) + \mu_1 s\tilde{V}(n,s) + \mu_4\tilde{V}(n,s) = Q_1\left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{21}$$

After simplification and rearrangement, we obtain the simple algebraic equation given by

$$\tilde{V}(n,s) = Q_1\left[\frac{1}{s^2 + \mu_1 s + \mu_4}\right]\left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{22}$$

which is further simplified to give

$$\tilde{V}(n,s) = Q_1\left[\frac{1}{(s + \mu_5)^2 + \gamma^2}\right]\left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right] \tag{23}$$

where

$$\gamma^2 = \mu_4 - (\mu_5)^2, \quad \mu_5 = \left(\frac{\mu_1}{2}\right) \tag{24}$$

At this juncture, in order to obtain the Laplace inversions of (23), we set

$$F(\mathrm{s}) = \left[\frac{1}{(s + \mu_5)^2 + \gamma^2}\right]$$

and

$$G(\mathrm{s}) = \left[\frac{\theta_n}{s^2 + {\theta_n}^2}\right]$$

so that the Laplace inversion of (23) is the convolution of $F(\mathrm{s})$ and $G(\mathrm{s})$ defined by (15) namely

$$F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{25}$$

Noting that

$$\mathcal{L}^{-1}[F(\mathrm{s})] = \frac{1}{p}\exp\ (-\mu_5 t)\sin(\gamma t) \tag{26}$$

and

$$\mathcal{L}^{-1}[G(\mathrm{s})] = \sin(\theta_n t) \tag{27}$$

Now using (26) and (27) in (25), (23) becomes

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$$V(n,t) = \frac{Q_1 e^{-\mu_5 t}}{\gamma(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\{\omega_2[\gamma e^{\mu_5 t}\sin\theta_n t - \theta_n \sin\gamma t] + \omega_0[\gamma e^{\mu_5 t}\sin\theta_n t + \theta_n \sin\gamma t] - \mu_1\gamma\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\gamma t]\} \quad (28)$$

where,
$\omega_1 = (\gamma + \theta_n)^2, \ \ \omega_2 = (\gamma - \theta_n)^2, \ \ \omega_0 = -(\mu_5)^2$

Thus, in view of (11), one obtains

$$V(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{Q_1 e^{-\mu_5 t}}{\gamma(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\{\omega_2[\gamma e^{\mu_5 t}\sin\theta_n t - \theta_n \sin\gamma t] + \omega_0[\gamma e^{\mu_5 t}\sin\theta_n t + \theta_n \sin\gamma t] - \mu_1\gamma\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\gamma t]\}\sin\frac{n\pi x}{l} \quad (29)$$

which represents the transverse displacement of an elastically supported prestressed shear beam when subjected to moving load traveling at a constant velocity.

Now, using (29) in (17), we have

$$\phi(n,t) = \frac{\mu_0 Q_1 e^{-\mu_5 t}}{\gamma(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\{\omega_2[\gamma e^{\mu_5 t}\sin\theta_n t - \theta_n \sin\gamma t] + \omega_0[\gamma e^{\mu_5 t}\sin\theta_n t + \theta_n \sin\gamma t] - \mu_1\gamma\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\gamma t]\} \quad (30)$$

Similarly, in view of (13), one obtains

$$\phi(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{\mu_0 Q_1 e^{-\mu_5 t}}{\gamma(\omega_1 - \omega_0)(\omega_2 - \omega_0)}\{\omega_2[e^{\mu_5 t}\sin\theta_n t - \theta_n \sin\gamma t] + \omega_0[\gamma e^{\mu_5 t}\sin\theta_n t + \theta_n \sin\gamma t] - \mu_1\gamma\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\gamma t]\}\cos\frac{n\pi x}{l} \quad (31)$$

which represents the rotation of an elastically supported prestressed shear beam when subjected to a moving load traveling at a constant velocity.

RESULTS AND DISCUSSION

The uniformly prestressed shear beam of lengths (L) = 50m, 55m, 60m and 65m respectively are considered in order to illustrate the analysis presented in this study. Similarly, the load is assumed to travel along the beam with different load speeds (c) = 40 m/s, 45 m/s, 50 m/s and 55 m/s respectively. The Young modulus of elasticity $E = 2.10924 \times 10^9 N/m^2$, moment of inertia $I = 2.87698 \times 10^{-3} m^4$, $\pi = 22/7$, the axial force $N = 4000N$, foundation stuffiness $K = 4000N/m^3$, Shear modulus $G = 4000N/m^3$ and the mass per unit length of the beam $M = 2758.291 kg/m$. The values of damping coefficient C_o are varied between 0 *and* 300.
In this present study, three special cases of the effect of damping coefficient C_o on dynamic response of a simply supported prestressed shear beam under the action of moving load were investigated. The cases are termed;

1. the effect of damping coefficient C_o on transverse displacement and rotation of a prestressed shear beam when the lengths of the beam (L) are 50m, 55m, 60m and 65m respectively,
2. the effect of damping coefficient C_o on transverse displacement and rotation of a prestressed shear beam when the load speeds (c) are 40 m/s, 45 m/s, 50 m/s and 55 m/s respectively and
3. the effect of damping coefficient C_o on critical velocity.

The transverse displacement $V(x,t)$ and rotation $\phi(x,t)$ of the beam are calculated and plotted against time t for various values of damping coefficient C_o. The results are shown on the various graphs given below.

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Figures 1 to 8 illustrate the transverse displacement and rotation of a simply supported prestressed shear beam subjected to a moving load traveling at a constant velocity, for various values of the damping coefficient C_o, with beam lengths (L) of 50m, 55m, 60m, and 65m respectively, while other parameters remain constant. It is evident from figures 1 to 8 that as the damping coefficient C_o increases, there is a significant decrease in both the transverse displacement and rotation of the beam. Consequently, increase in the value of damping coefficient C_o reduces the amplitude of vibrations and oscillations of the vibrating beam. Practically speaking, the damping mechanisms absorb and effectively dissipate energy that would otherwise keep it oscillating, causing the beam to return to equilibrium more quickly and with less overshoot. Hence, the presence of damping coefficient C_o increases the overall stability of the beam system.

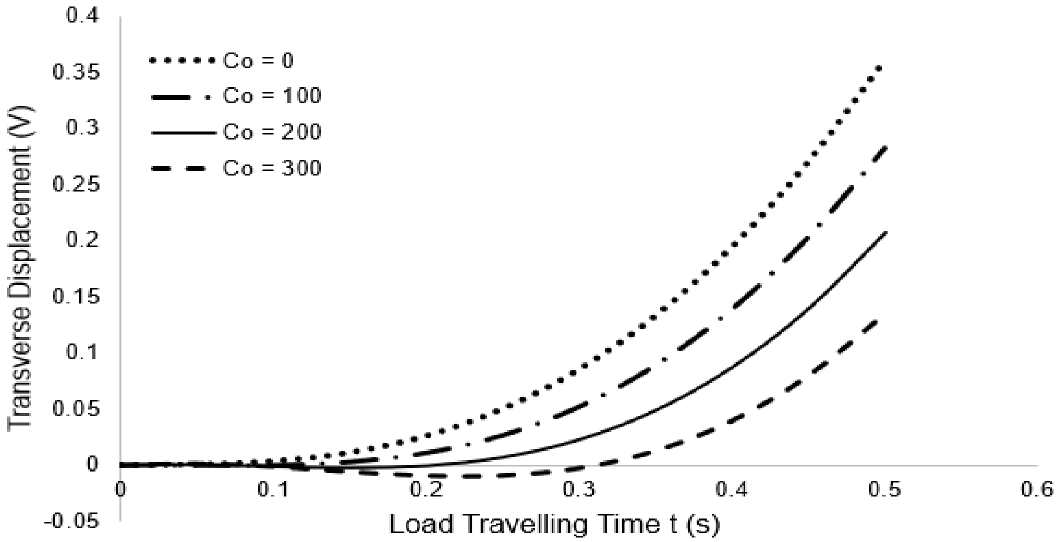

Figure 1: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 50 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

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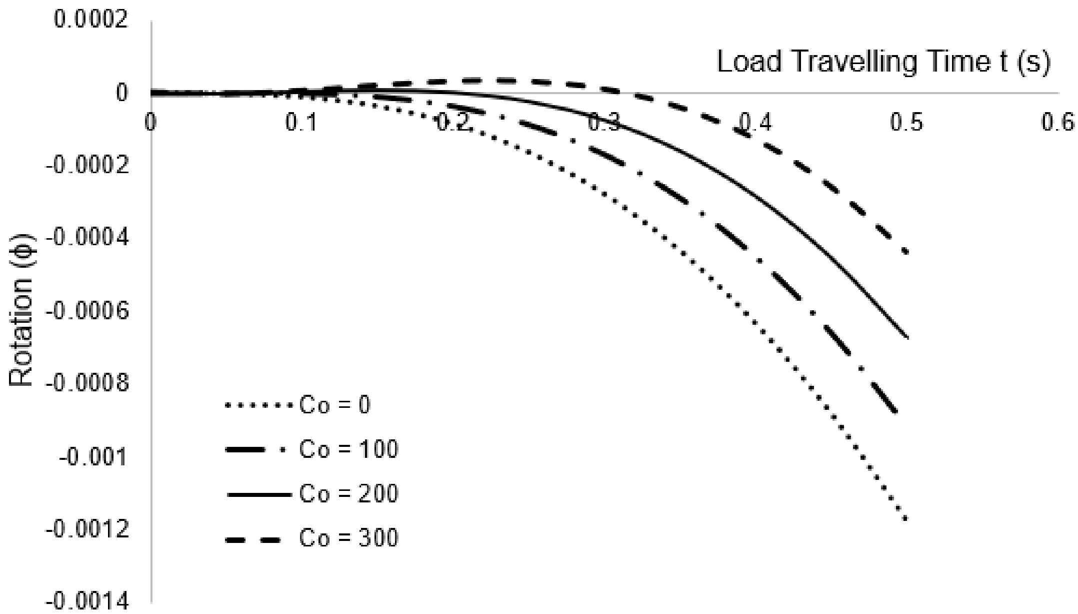

Figure 2: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 50 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

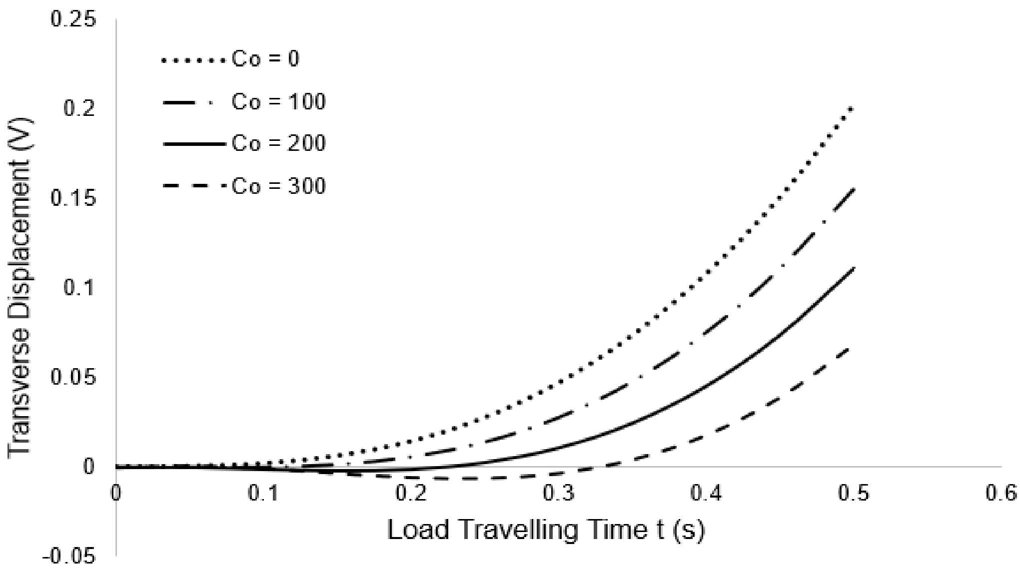

Figure 3: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 55 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

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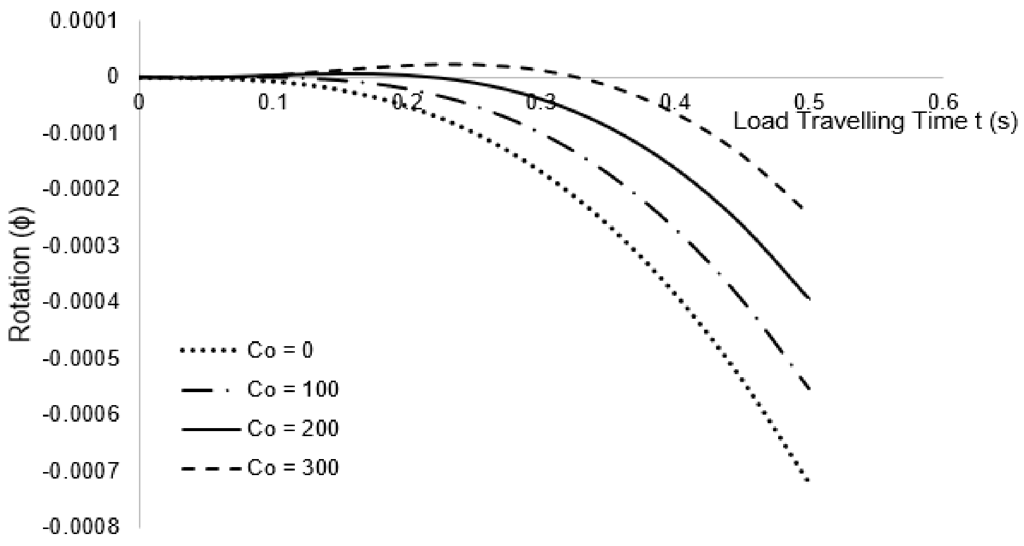

Figure 4: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 55 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

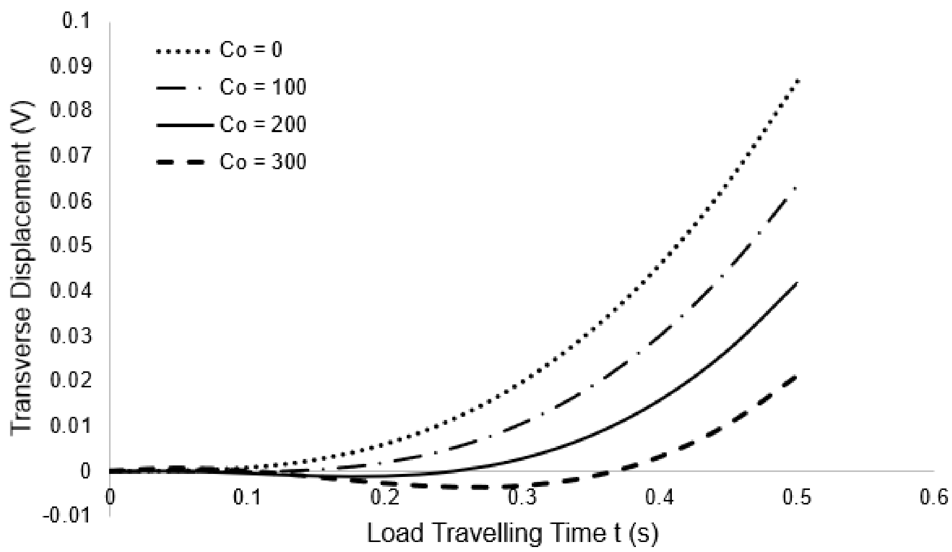

Figure 5: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 60 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

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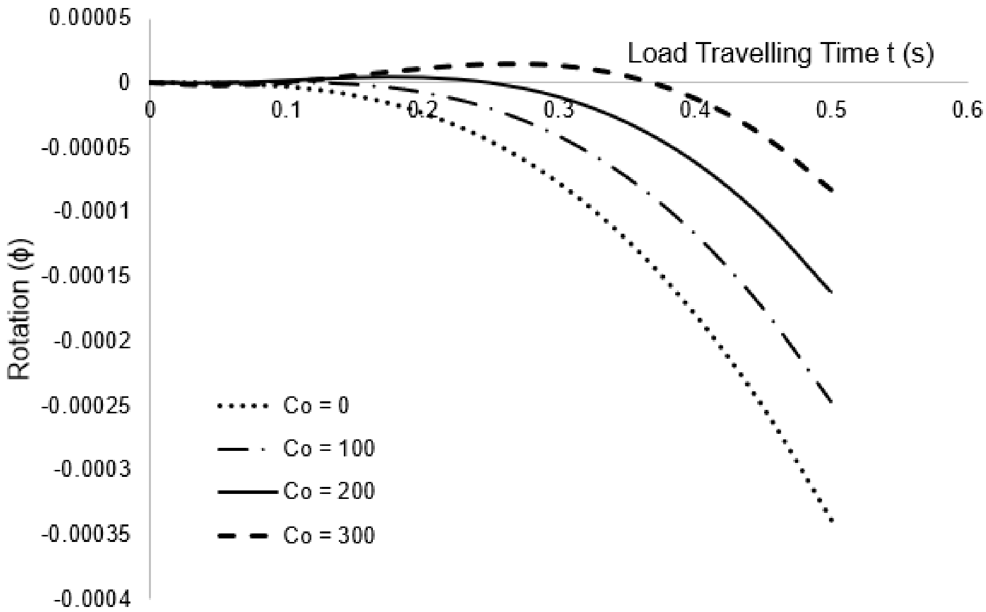

Figure 6: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 60 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

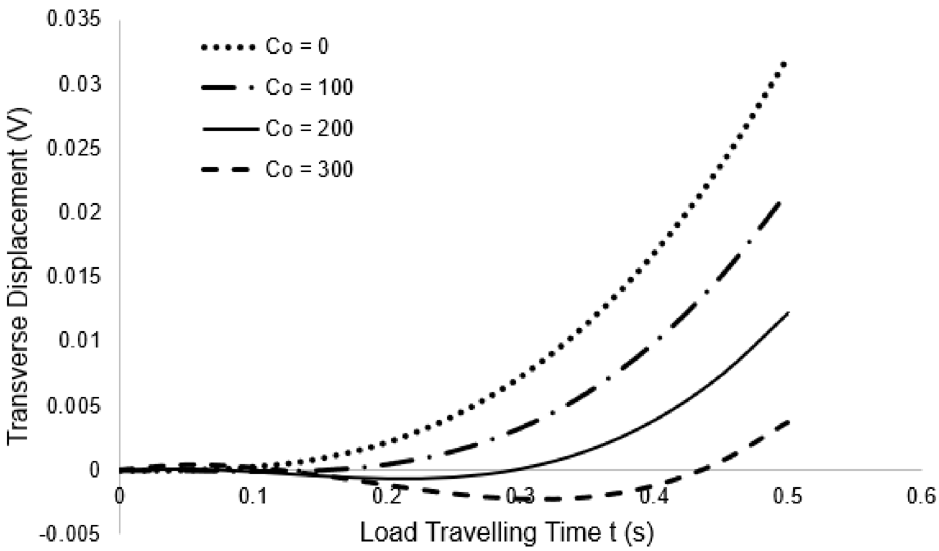

Figure 7: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 65 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

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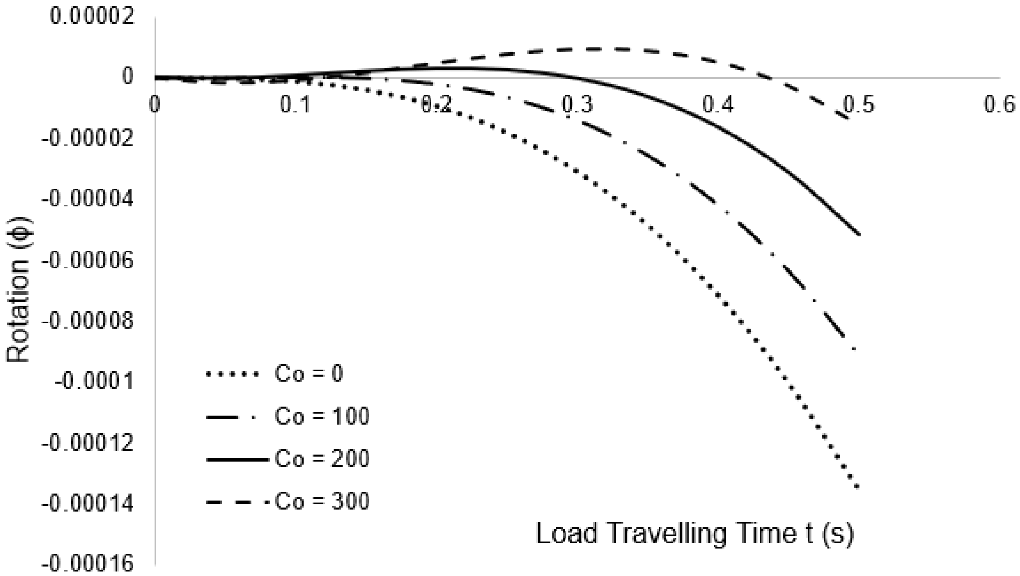

Figure 8: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the beam length L = 65 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ C = 40$

In a similar manner, the response amplitude profile of a simply supported uniformly prestressed shear beam subjected to moving load for various values of damping coefficient C_o, with load speeds (c) set at 40 m/s, 45 m/s, 50 m/s and 55 m/s respectively and for the fixed values of other parameters are presented in figures 9 to 16. The graphs clearly indicate that as the velocity of the moving load increases, the dynamic effects become increasingly significant, resulting in greater transverse displacement and rotation of the beam. However, increase in the value of damping coefficient C_o reduces the transverse displacement and rotation of the vibrating beam considerably. In practical applications, this implies that as the value of damping coefficient C_o increases, the vibration amplitude and frequency of vibrations of the beam reduce significantly. Consequently, the beam exhibits enhanced rigidity and stability, enabling it to withstand lateral deflection and severe vibrations, thus allowing it to support larger transverse loads even at higher velocities without the risk of buckling. Hence, the potential for flexural cracking or bending within the beam system is significantly reduced with the presence of damping.

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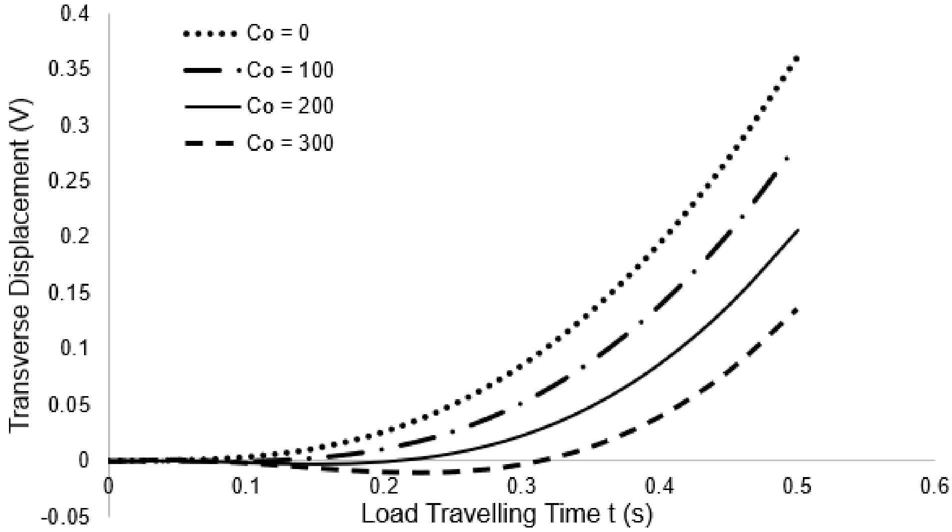

Figure 9: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 40 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

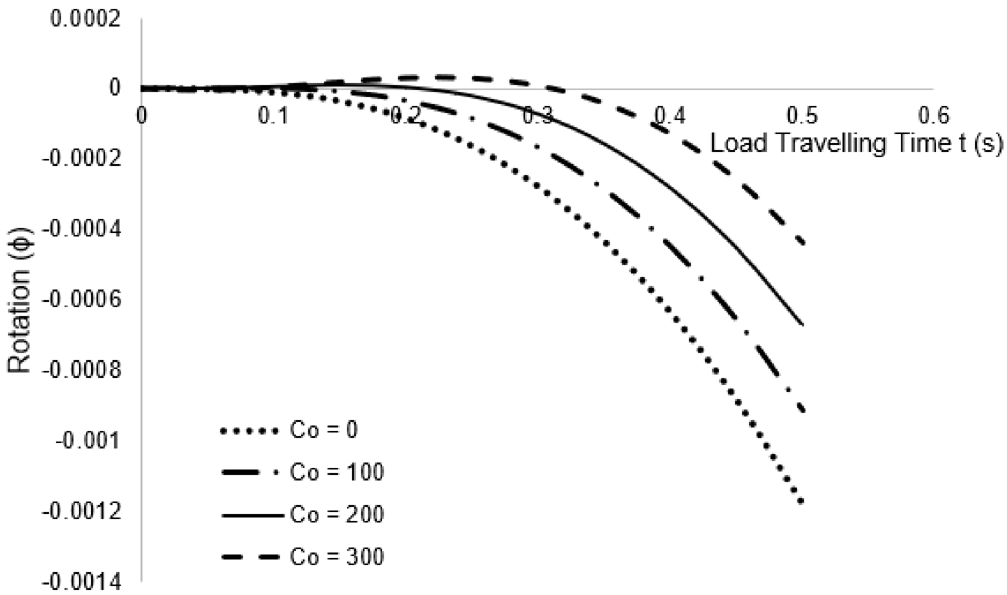

Figure 10: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 40 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

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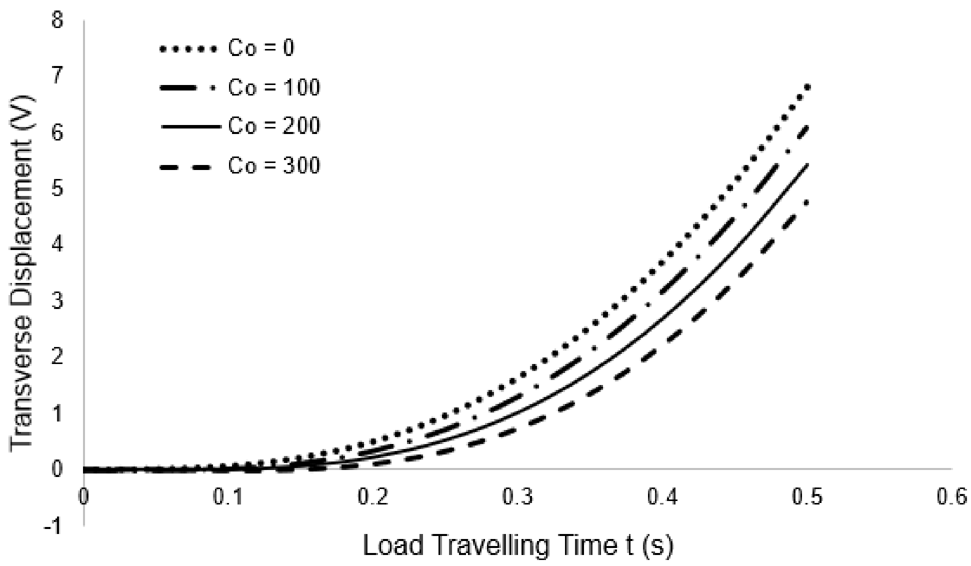

Figure 11: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 45 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

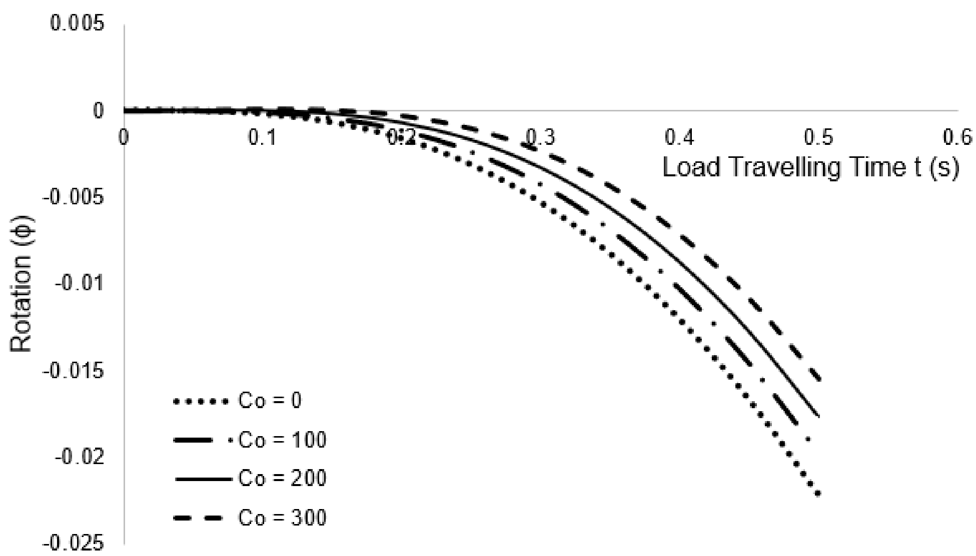

Figure 12: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 45 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

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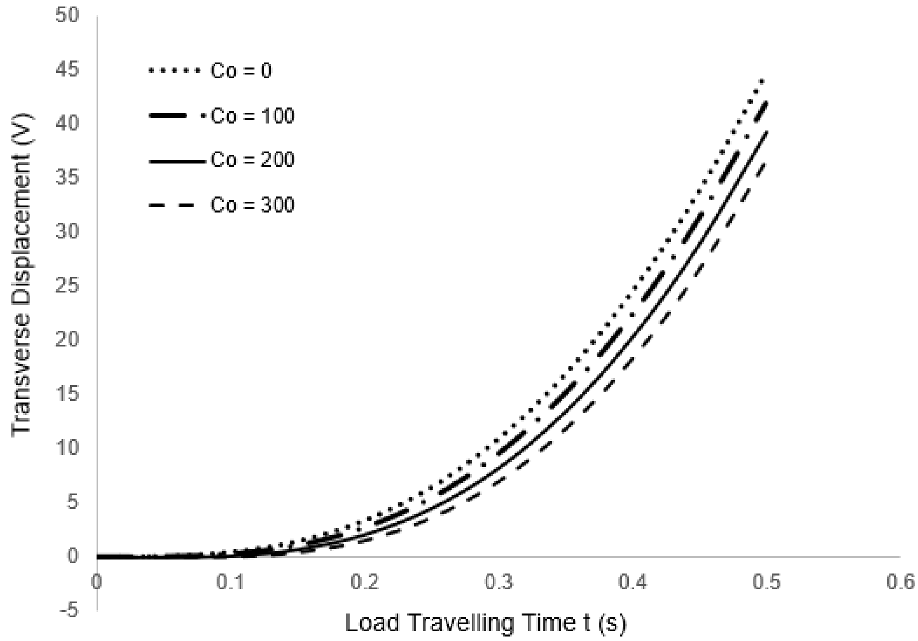

Figure 13: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 50 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

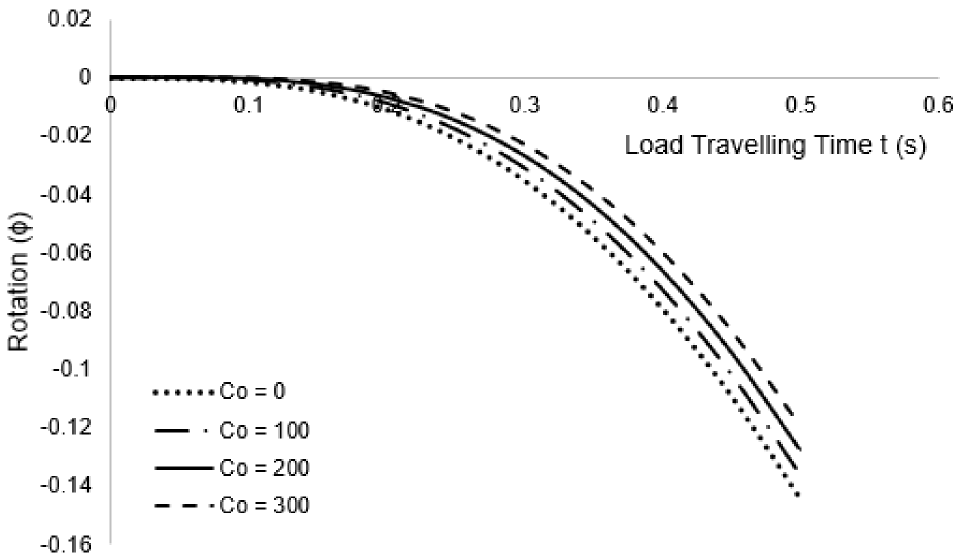

Figure 14: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 50 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

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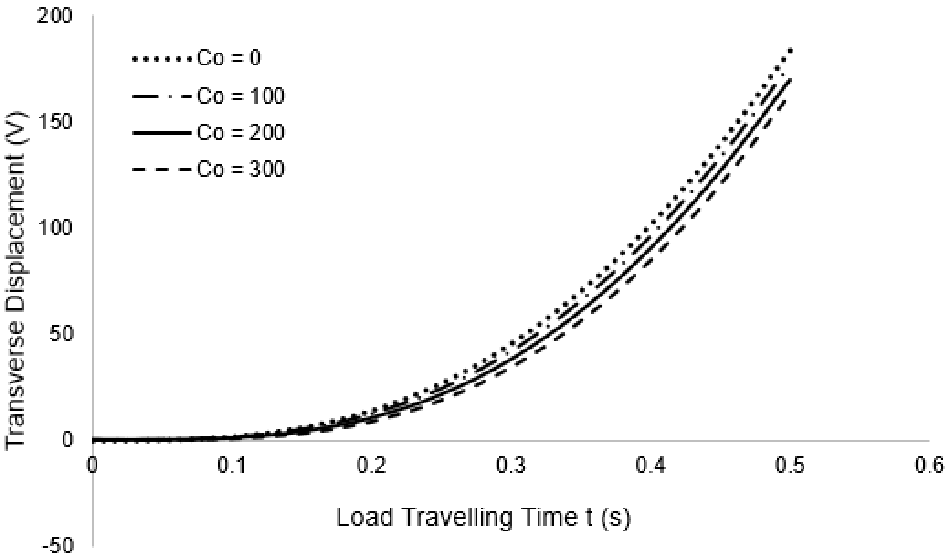

Figure 15: Transverse displacement of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 55 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

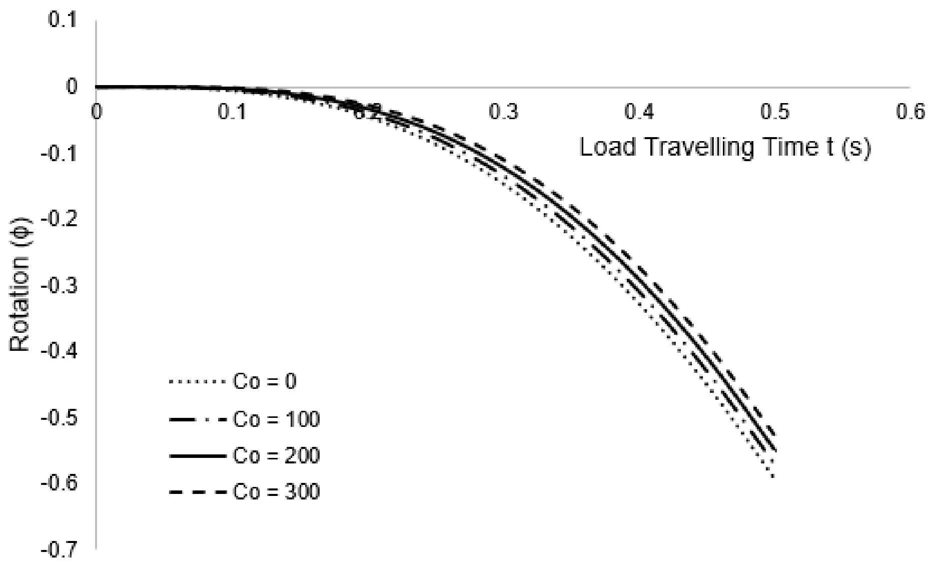

Figure 16: Rotation of a simply supported prestressed shear beam under the action of moving load for various values of damping coefficient C_o when the Load Speed C = 55 and for fixed values of $K = 4000,\ G = 4000,\ N = 4000\ and\ L = 50$

Finally, the effect of damping coefficient C_o on the critical velocity of a simply supported prestressed shear beam traversed by moving load is presented in figure 17. It is observed from the graph that for various values of damping coefficient C_o and for the fixed values of other parameters, the higher the value of the damping coefficient C_o, the higher the critical velocity of the beam. In practical terms, increase in damping coefficient C_o reduces the peak amplitude of resonance, where the beam's natural frequency matches the excitation frequency. Consequently,

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damping contributes to the dynamic stability of the beam system, mitigating the risk of resonance that may result in structural failure and thus, safeguarding the safety of the structure's occupants.

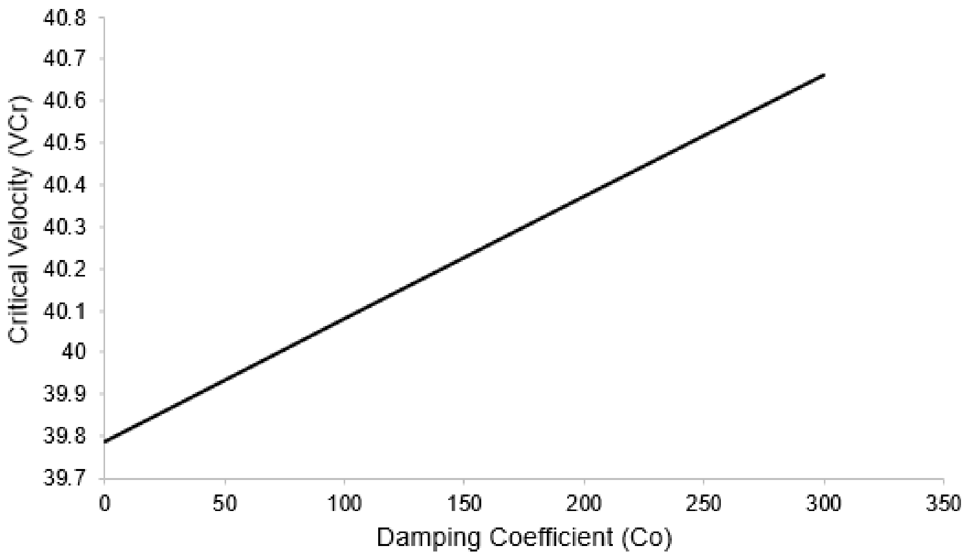

Figure 17: Variation of the critical velocity (VCr) against damping coefficient (C_o)

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CONCLUSION

This paper examines damping coefficient effect on transverse displacement and rotation under moving of a prestressed shear beam supported by an elastic foundation. A solution methodology that incorporates finite Fourier transform techniques, Laplace transformation, and convolution theory is utilized to obtain the solution for the coupled second-order partial differential equations that define the motion of the beam-load system. Comprehensive analyses are conducted to examine the effect of damping coefficient on the transverse displacement and rotation of prestressed shear beams of different length sizes when subjected to a moving load traversing at different velocities. Additionally, the study explores how the damping coefficient affects the critical velocities of the vibrating system. The graphs plotted clearly illustrate that the damping coefficient significantly enhances the stability of the beam under the moving load. The findings reveal that both the transverse displacement and rotation of the beam are noticeably reduced as the damping coefficient increases. Additionally, it is observed that higher damping coefficient values correspond to increased critical velocities, indicating a more robust dynamic system. Therefore, in the design of engineering structures such as bridges, pipelines, railway tracks, aerospace components, railway bridges, overhead cranes, cableways, and tunnels, it is indispensable to comprehend the influence of damping in order to design structures capable of withstanding the severe effects of dynamic loads and vibrations induced by environmental factors, thereby ensuring the safety, reliability, and efficiency of the design.

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