Vol-05 Issue 07, July-2021 ISSN: 2456-9348
Impact Factor: 5.004

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International Journal of Engineering Technology Research & Management

ON HOMOGENEOUS CUBIC EQUATION WITH FOUR UNKNOWNS $x^3 - y^3 = 4(w^3 - z^3) + 6(x - y)^3$

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Abstract

This paper concerns with the problem of obtaining non-zero distinct integer solutions to homogeneous cubic equation with four unknowns given by $x^3 - y^3 = 4(w^3 - z^3) + 6(x - y)^3$. A few interesting properties among the solutions are presented.

Keywords:

homogeneous cubic, cubic with four unknowns ,integer solutions

Notation:

$$t_{m,n} = n(1 + \frac{(n-1)(m-2)}{2})$$

Introduction:

The cubic diophantine equations are rich in variety and offer an unlimited field for research [1,2]. In particular refer [3-24] for a few problems on cubic equation with 3 and 4 unknowns. This paper concerns with yet another interesting non-homogeneous cubic diophantine equation with four unknowns given by $x^3 - y^3 = 4(w^3 - z^3) + 6(x - y)^3$ for determining its infinitely

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many non-zero distinct integral solutions by employing the linear transformations .A few interesting relations among the solutions are presented.

Method of analysis:

The homogeneous cubic equation with four unknowns to be solved is

$$x^3 - y^3 = 4(w^3 - z^3) + 6(x - y)^3 \quad (1)$$

Introduction of the linear transformations

$$x = 6u + 2v, y = 6u - 2v, w = 3s + 2v, z = 3s - 2v \quad (2)$$

in (1) leads to the well-known Pythagorean equation

$$u^2 = v^2 + s^2 \quad (3)$$

Employing the most cited solutions of the Pythagorean equation and using (2),

one obtains the following two sets of integer solutions to (1):

Set:1

$$x = x(p,q) = 6(p^2 + q^2) + 4pq \qquad y = y(p,q) = 6(p^2 + q^2) - 4pq$$

$$w = w(p,q) = 3(p^2 - q^2) + 4pq \qquad z = z(p,q) = 3(p^2 - q^2) - 4pq$$

Properties :

1. Each of the following expressions is a nasty number:
   $2(y + w), 2(x + z), x + y + z + w$ when $p = 2rs, q = 3r^2 - s^2$
2. When $q = 1, 3(x + w + z) - 24t_{3,p}$ is a nasty number.
3. Each of the following expressions is expressed as difference of two squares:
   $3(y + w + z), 3(x + w + z), 3(x + 2w), (xy - wz)$

Set:2

$$x = x(p,q) = 8p^2 + 4q^2 \qquad y = y(p,q) = 4p^2 + 8q^2$$

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$$w = w(p,q) = 2(p^2 - q^2) + 6pq \qquad z = z(p,q) = -2(p^2 + q^2) + 6pq$$

Properties:

1. $3(x + y - z + w)$ is a nasty number when $p = 2rs, q = 2r^2 - s^2$

2. Each of the following expressions is expressed as difference of two squares

$3(y + 4w), 3(x + 2w), 3(y + 2z), 3(x - y + 2w - 2z)$

3. When $q = 1, x + 2w = 24t_{3,p}$

4. When $p = 1, y + 2z = 24t_{3,q}$

Set:3

Write (3) as

$$v^2 + s^2 = u^2 * 1 \tag{4}$$

Assume

$$u = p^2 + q^2 \tag{5}$$

Consider 1 as

$$1 = i * (-i) \tag{6}$$

Substituting (5) & (6) in (4) and employing the method of factorization ,define

$$v + is = i * (p + iq)^2$$

On equating the real and imaginary parts,one obtains

$$v = -2pq, s = p^2 - q^2 \tag{7}$$

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Using (5) and (7) in (2) ,the corresponding integer solutions to (1) are obtained as below:

$x = x(p,q) = 6(p^2 + q^2) - 4pq$ $y = y(p,q) = 6(p^2 + q^2) + 4pq$

$w = w(p,q) = 3(p^2 - q^2) - 4pq$ $z = z(p,q) = 3(p^2 - q^2) + 4pq$

Properties:

1. $2(x+y)$ is a nasty number when $p = 2rs, q = r^2 - s^2$
2. $18(x+y)$ is a cubical integer when $p = m(m^2 + n^2), q = n(m^2 + n^2)$
3. $3(x+y)$ is written as the sum of two squares
4. $6(w+z)$ is written as the difference of two squares

5 . When $q = 1, x + w = t_{20,p} + 3$

6. When $p = 1, x - z = t_{20,q} + 3$

7. When $p = 1, y - 2w = 24t_{3,q}$

Conclusion:

In this paper, an attempt has been made to obtain many non-zero distinct integral solutions to the cubic equation with four unknowns given by $x^3 - y^3 = 4(w^3 - z^3) + 6(x-y)^3$ through employing linear transformations .The researchers may search for other choices of linear transformations for finding integer solutions to the considered cubic equation.

As cubic equations are rich in variety, the readers may search for obtaining integer solutions to other choices of cubic equations with multivariables.

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