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KEPLER'S LAWS - THREE PATTERNS IN THE MOTION OF THE PLANETS

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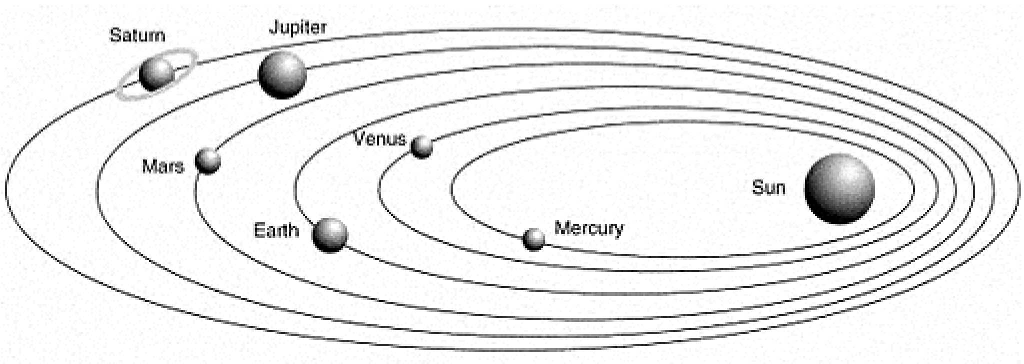
Abstract:
Kepler's laws of planetary motion are a set of three laws describing the motion of planets around the Sun. German mathematician and astronomer Johannes Kepler published these laws between 1609 and 1619 based on observations made by his mentor, Danish astronomer Tycho Brahe. Although Kepler applied the three laws to planets in the solar system, they can be extended to planets outside the solar system, asteroids, and artificial satellites. These laws are collectively known as First Law, Second Law, and Third Law.
Key words: planetary, solar system, planets,satellites,asteroids

Description

The 17th century astronomer Johannes Kepler pointed out three patterns in the motion of the planets. They're called Kepler's laws. Kepler's first law is that the planets trace out ellipses as they go around the Sun. The orbits look like circles, but they're not. They're slightly flattened into ovals, and even the Sun is off-cente Kepler's laws of planetary motion are a set of three laws describing the motion of planets around the Sun. German mathematician and astronomer Johannes Kepler published these laws between 1609 and 1619 based on observations made by his mentor, Danish astronomer Tycho Brahe. Although Kepler applied the three laws to planets in the solar system, they can be extended to planets outside the solar system, asteroids, and artificial satellites. These laws are collectively known as First Law, Second Law, and Third Law.



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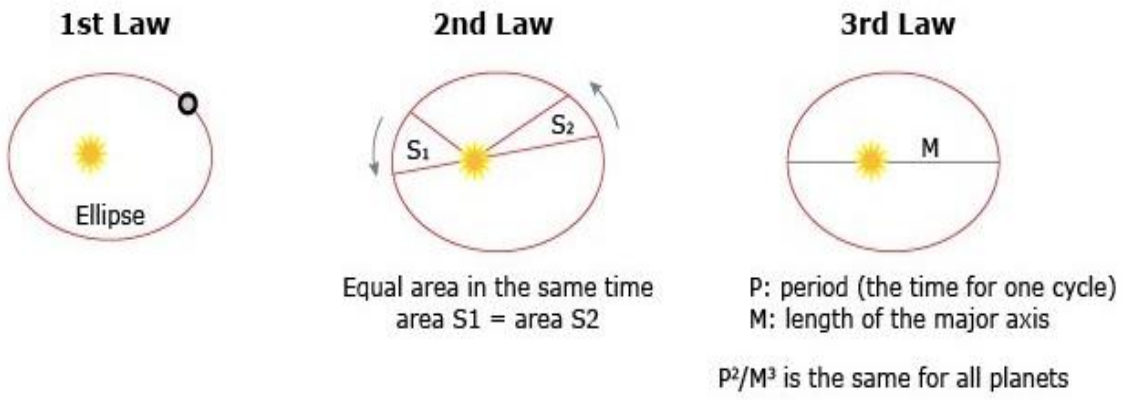


1.First Law

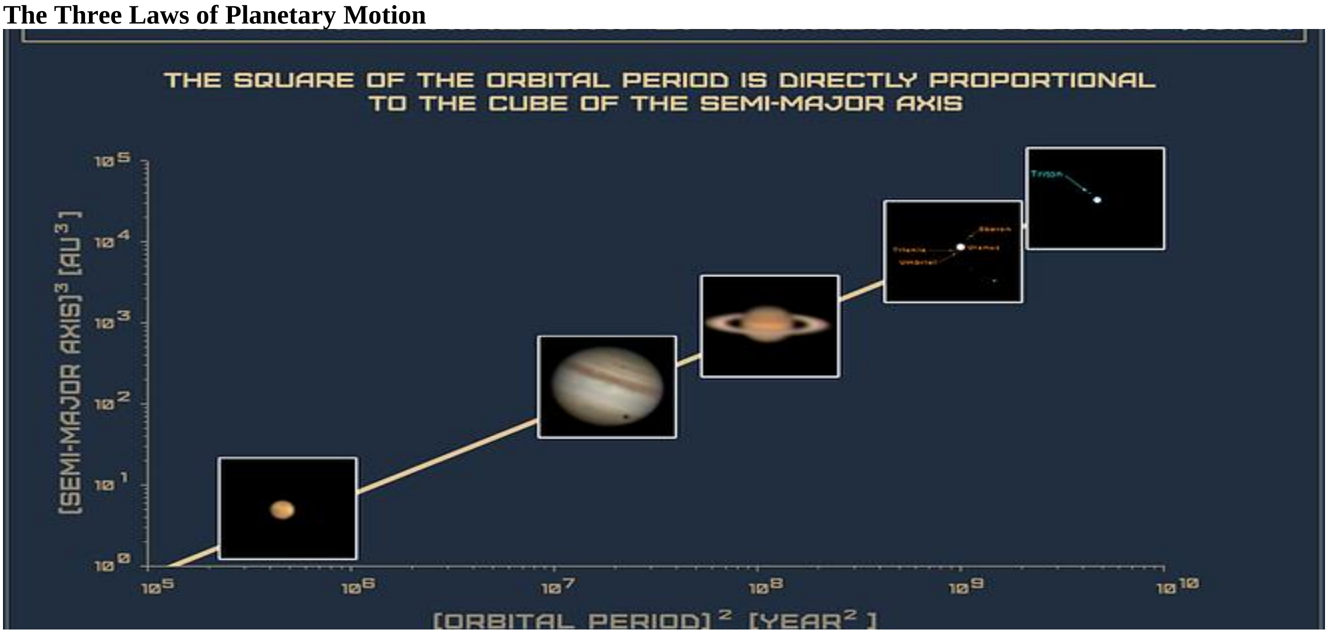
Statement: "All planets orbit around the Sun in a path described by an ellipse with the Sun at one of its two foci".

Also known as the Law of Ellipses, Kepler concluded that all solar system planets have elliptical orbits. The Sun's center is at one of the foci. When a planet revolves around the Sun, its distance from the Sun constantly changes. The point of the closest approach to the Sun is the perihelion, and the furthest point is the aphelion. Kepler's first law is used to study the trajectories of planets, asteroids, and comets by applying the ellipse equations.

The final post in a series of three covering Kepler's Laws of Planetary Motion. In this post we'll look at law 3 or 'The Law of Periods' – The square of the period of any planet is proportional to the cube of the semimajor axis of it's orbit.

Johannes Kepler's Third Law of Planetary Motion, also known as the "harmonic law," is a fundamental principle that relates the period of a planet's orbit to its distance from the sun. This law helped to revolutionize our understanding of the solar system and has been a cornerstone of astronomy ever since.

The Three Laws of Planetary Motion



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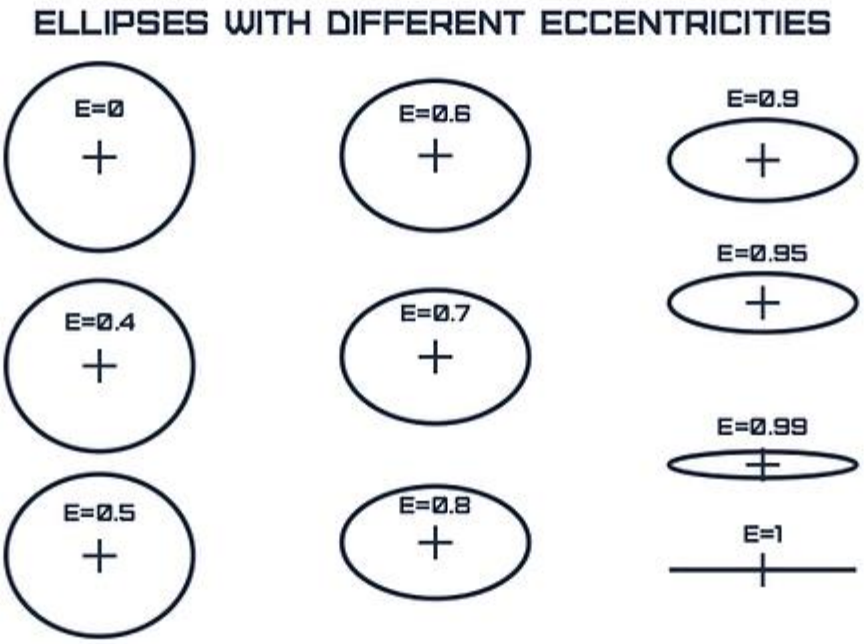
Kepler's Laws of Planetary Motion
Ellipses

To be able to understand Kepler's laws, let's first quickly look at the geometry of ellipses. Ellipses are curves that can be described by two focus points, or focal points. A circle is a particular case of an ellipse with only one focus point at its center, where the distance between it and every point on the circle's perimeter is the same. Ellipses look like a squashed circle. The way that ellipses are described is by the quantity called eccentricity (E). A shape's eccentricity defines the kind of conic section it is: zero for a circle, zero to one for an ellipse, one for a parabola, and greater than one for a hyperbola.



How the eccentricity affects the shape of an ellipse

Take a look at the diagram below with ellipses with an eccentricity of 0.6. The leftmost ellipse in the diagram has the locations of its center and two focal points marked. The foci are at unique locations inside an ellipse. The sum of the distances from each focus to any other point on the ellipse is the same for a given ellipse. The semi-major (SMJA) and semi-minor axes (SMNA) are the longest and shortest distances from the ellipse's center to the perimeter. The apoapsis (A) and periapsis (P) are the longest and shortest distances from one focal point to the perimeter along the semi-major axis.

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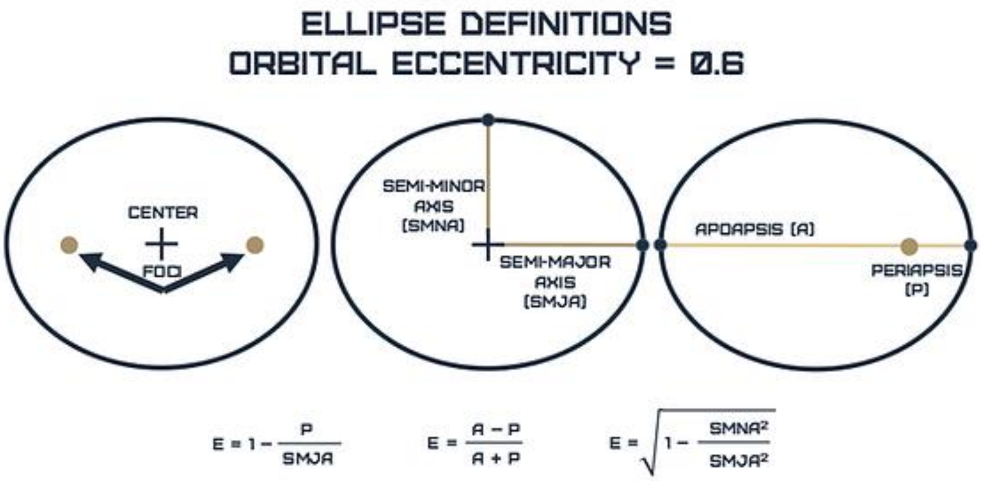

The definitions of ellipticity, semi-major axis, semi-minor axis, periapsis, and apoapsis.
There are many geometrical relationships between the eccentricity, semi-major axis, semi-minor axis, periapsis, and apoapsis; the diagram above contains a few equations of these relationships. The two foci points converge to the center when an ellipse loses ellipticity and becomes a circle. Not only that, notice how the semi-major axis, semi-minor axis, periapsis, and apoapsis all become the same length for circles.

Kepler's First Law
Kepler’s first law states that the orbital path of a planet around the Sun is an ellipse with the Sun at one of the focal points. The early geocentric and heliocentric models of the Solar System break this law because they assumed the planets had circular orbits.
Kepler’s First Law. The orbital path of an object around a body is shaped like an ellipse with the body at one of the two foci. This orbital path has an eccentricity of 0.4. Image by John H Boisvert for Slooh
This law tells us that the distance between the planet and the Sun is not constant. A planet is at its closest approach at perihelion and its furthest point at aphelion. Note that these terms are similar to periapsis and apoapsis except with different suffixes. The suffix of peri-/apo- changes depending on the focal point (see table below), but the geometric meaning remains the same. Also note that, although the diagram shows a dramatic eccentricity of 0.4, most planets in the solar system have very low eccentricities. In other words, their orbital paths are nearly, but not precisely, circular.
You can observe Kepler’s first law in action when looking at the Earth-Moon system. The Moon orbits the Earth in an ellipse, with the Earth at a focal point. That means that the distance between the Earth and Moon is continuously changing. A supermoon occurs when a Full Moon is near perigee, and a micromoon happens when the Full Moon is near apogee.

Kepler's Second Law
Kepler’s second law states that a line from a planet to the Sun sweeps out equal areas during equal time intervals.
The amount of time it takes for the planet to travel the perimeter of each shaded region is the same, and the area that those journeys carve out is also the same. This law means that the planet must be moving slower when traveling along the perimeter of the shaded region on the left than along the right region. In general, a planet slows down when it nears apoapsis, and it speeds up as it nears periapsis.

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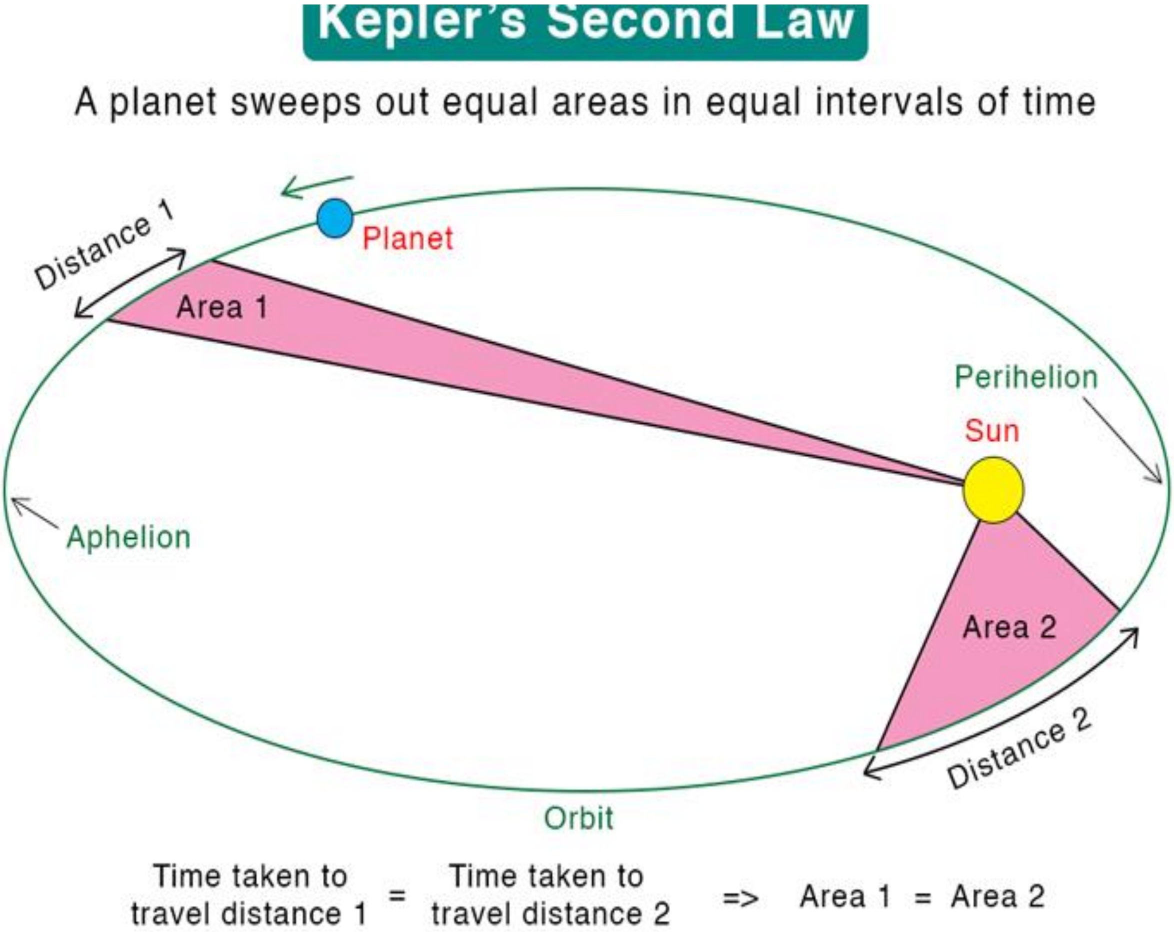

Kepler's Second Law. The area of both shaded regions, "A," and the length of time the orbiting object takes along each arc is the same. Image by John H. Boisvert for Slooh

Kepler's Third Law

Kepler's third law states that the square of the orbital period is directly proportional to the cube of the semi-major axis of its orbit. Writing this in equation form gives the following:

$$p^2 = Ca^3$$

In the equation, "p" is the orbital period (the time it takes the object to complete a full orbit), "a" is the semi-major axis, and "C" is the constant of proportionality. The constant is the same for every planet in the solar system. Graphically, this means that, when the square of the orbital period and the cube of the semi-major axis are plotted, they should form a linear graph, as shown below.

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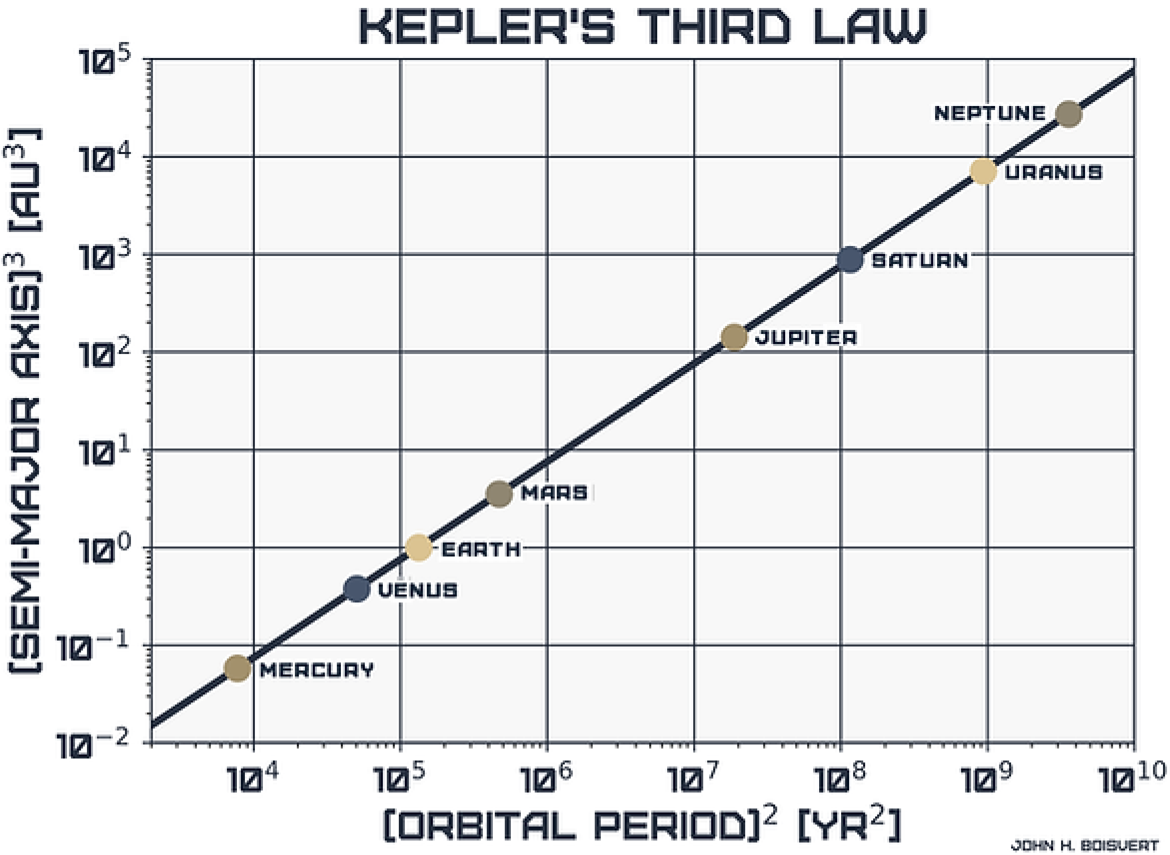

A plot of the square of the orbital period vs. the cube of the semi-major axis for the Solar System planets.

The wandering planets follow the same physical laws as all matter in the universe. In this quest, you will discover how humanity unveiled the extraordinary physical laws of orbital motion. You'll learn about Johannes Kepler and his three laws of planetary motion. You'll also learn about Sir Isaac Newton and his theory of gravitation. By the end of the quest, you will discover the similarities between Kepler's laws and Newton's gravity, and how these laws apply to our tiny corner of the galaxy—the Solar System.

Slooh's Online Telescope is a learning platform designed to support any educator in teaching astronomy to meet NGSS requirements by collecting and analyzing real-world phenomena. No previous experience with telescopes is necessary to quickly learn how to use Slooh to explore space with your students.

Kepler's Third Law states that the square of the orbital period of a planet is proportional to the cube of its average distance from the sun. In other words, **if you square the time it takes a planet to complete one orbit around the sun and divide it by the cube of its distance from the sun, you will always get the same value for all the planets in our solar system**.

This law was a significant departure from the traditional belief at the time that planets moved in perfect circles around the sun at a constant speed. Kepler's observations of the motion of planets had led him to conclude that their speed changed as they moved through their elliptical orbits, and their period depended on their distance

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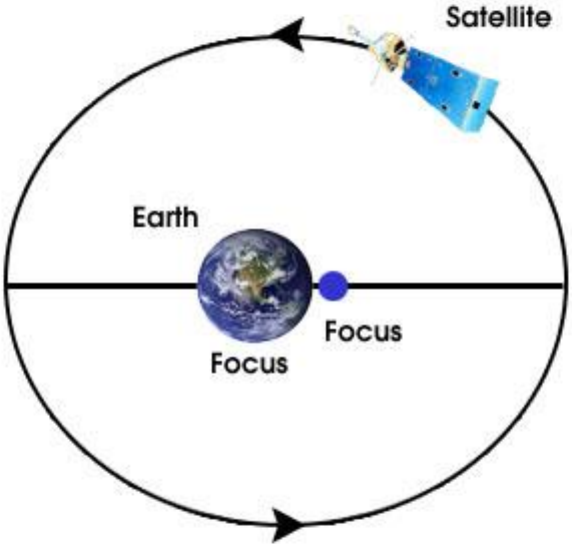
from the sun. He then extended his analysis to other planets and found that their orbital periods followed this



same relationship.
Kepler's Third Law was crucial in determining the relative distances of the planets from the sun. By measuring the orbital periods of the planets and knowing their average distances from the sun, astronomers could calculate the distance of any planet from the sun. This allowed astronomers to construct accurate models of the solar system and paved the way for further research into the nature of the universe.
One of the most significant implications of Kepler's Third Law was that it helped establish a relationship between the size of a planet's orbit and its distance from the sun. The farther a planet is from the sun, the longer it takes to complete one orbit. This relationship also helps explain why the outer planets in our solar system are much larger than the inner planets.
Neptune – The predicted planet
Kepler's Third Law also played a critical role in the discovery of Neptune. In the mid-19th century, astronomers noticed that the orbit of Uranus did not follow the predicted path, suggesting that there was another planet beyond it. Using Kepler's Third Law, astronomers were able to calculate the position of this hypothetical planet, and in 1846, Neptune was discovered in the predicted location.
Today, Kepler's Third Law remains a critical tool for astronomers in their study of the solar system and other celestial bodies. It allows us to calculate the distances of planets from their stars and has been used to discover hundreds of exoplanets in other solar systems.
Johannes Kepler's Third Law of Planetary Motion, or the harmonic law, is a fundamental principle that relates the period of a planet's orbit to its distance from the sun. This law helped to revolutionize our understanding of the solar system and has been a cornerstone of astronomy ever since. By establishing a relationship between the size of a planet's orbit and its distance from the sun, Kepler's Third Law has played a critical role in our understanding of the universe.
Kepler's Third Law Mathematics
As with the parts one and two of this series, it's not vital to understand the mathematics of the third law to gain a high level understanding of the concept, but again I've included it here for you to read through if you would like to.
Kepler's third law can be derived mathematically as follows:
Let **T** be the period of the planet's orbit, which is the time it takes to complete one revolution around the sun.
Let **a** be the semi-major axis of the planet's elliptical orbit, which is the longest radius of the ellipse.
We can express Kepler's third law as:

$$T^2 \propto a^3$$

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To derive this law, we start with Newton's law of gravitation, which states that the force of attraction between two objects is proportional to the product of their masses and inversely proportional to the square of their distance. For the case of a planet orbiting the sun, we can write:

$$F = \frac{GMm}{r^2}$$

where **G** is the gravitational constant, **M** is the mass of the sun, **m** is the mass of the planet, and **r** is the distance between them.
Since the force of gravity is the centripetal force that keeps the planet in orbit, we can equate it to the centripetal force:

$$F = m\left(\frac{v^2}{r}\right)$$

where **v** is the velocity of the planet.
Equating these two expressions for **F**, we get:

$$\frac{GMm}{r^2} = m\left(\frac{v^2}{r}\right)$$

Simplifying, we get:

$$v^2 = \frac{GM}{r}$$

Now, we can express the semi-major axis **a** in terms of **r** as:

$$a = \frac{r}{(1-e^2)}^{0.5}$$

where **e** is the eccentricity of the elliptical orbit, which is a measure of how much it deviates from a circle.
Substituting this expression for **r** into the equation for v^2, we get:

$$v^2 = \frac{GM}{(1-e^2)}r$$

We can rearrange this equation as:

$$T^2 = 4\pi^2\left(\frac{a^3}{GM}\right)$$

This is Kepler's third law in its final form, which states that the square of the orbital period of a planet is proportional to the cube of the semi-major axis of Johannes Kepler pointed out three patterns in the motion of the planets

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The Law of Harmonies

Kepler's third law - sometimes referred to as the **law of harmonies** - compares the orbital period and radius of orbit of a planet to those of other planets. Unlike Kepler's first and second laws that describe the motion characteristics of a single planet, the third law makes a comparison between the motion characteristics of different planets. The comparison being made is that the ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets. As an illustration, consider the orbital period and average distance from sun (orbital radius) for Earth and mars as given in the table below.

Planet	Period (s)	Average Distance (m)	T^2/R^3 (s^2/m^3)
Earth	3.156×10^7 s	1.4957×10^{11}	2.977×10^{-19}
Mars	5.93×10^7 s	2.278×10^{11}	2.975×10^{-19}

Observe that the T^2/R^3 ratio is the same for Earth as it is for mars. In fact, if the same T^2/R^3 ratio is computed for the other planets, it can be found that this ratio is nearly the same value for all the planets (see table below). Amazingly, every planet has the same T^2/R^3 ratio.

Planet	Period (yr)	Average Distance (au)	T^2/R^3 (yr^2/au^3)
Mercury	0.241	0.39	0.98
Venus	.615	0.72	1.01
Earth	1.00	1.00	1.00
Mars	1.88	1.52	1.01
Jupiter	11.8	5.20	0.99
Saturn	29.5	9.54	1.00
Uranus	84.0	19.18	1.00
Neptune	165	30.06	1.00
Pluto	248	39.44	1.00

(**NOTE**: The average distance value is given in astronomical units where 1 a.u. is equal to the distance from the earth to the sun - 1.4957×10^{11} m. The orbital period is given in units of earth-years where 1 earth year is the time required for the earth to orbit the sun - 3.156×10^7 seconds.)

Kepler's third law provides an accurate description of the period and distance for a planet's orbits about the sun. Additionally, the same law that describes the T^2/R^3 ratio for the planets' orbits about the sun also accurately describes the T^2/R^3 ratio for any satellite (whether a moon or a man-made satellite) about any planet. There is something much deeper to be found in this T^2/R^3 ratio - something that must relate to basic fundamental principles of motion. In the next part of Lesson 4, these principles will be investigated as we draw a connection between the circular motion principles discussed in Lesson 1 and the motion of a satellite.

Circles and Ellipses

Mathematically, a circle is defined as the set of all points that are the same distance from some chosen center. Put a pin in a corkboard, then take a piece of string and tie both ends to the pin, making a loop. Then, put a pen in the loop, stretch the string tight, and sweep around the board, drawing as you go. You've just made a circle, centered on the pin.

For an ellipse, you need 2 pins, and you tie the ends of the string to the pins, with some slack. And stretch the string tight with the pen, and sweep around the board, drawing as you go. What you get in that case is an ellipse. The 2 pins are the focus points, or foci, of the ellipse. For all the points on the ellipse, the distance to the first focus, plus the distance to the second focus, is a constant, that's the length of the string, in our construction.

The length of that string is also equal to the length of the long axis of the ellipse, the major axis. If we unpinned the string and straightened it out, it would reach exactly across the major axis. We conclude that for any point on an ellipse, the distance to one focus plus the distance to the other focus equals the length of the major axis.

Radii and Areas

To make a circle, we only needed to make one choice: the radius, which we'll call a. For an ellipse, the equivalent is the radius along the major axis, which we'll also label a, and we'll call that the semi-major axis. But with an ellipse, we also have to choose the distance between the center and either focus.

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We can choose whatever distance we want, as long as it's smaller than a. Tradition dictates that we express that distance as a times e, where e is a number smaller than one, which is called the eccentricity. When e is zero, the foci coincide at the center, and we go back to a circle of radius a; as e gets larger, and closer to one, the foci separate and we get a more elongated ellipse.

For a circle, we know area equals pi a-squared. For an ellipse, it turns out to be pi a-squared times the square root of one minus-e-squared.

A Coordinate System

To understand the mathematical equation for an ellipse, in polar coordinates, let's start by introducing a coordinate system. We'll put the origin at one of the foci, and we'll lay down x and y axes along the major and minor axes. To specify the points on the ellipse, we would use polar coordinates: r is the distance from the origin, and theta is the angle measured counter-clockwise from the x axis.

First, let's think about what we expect. It's going to be a rising and falling function, for theta equals zero, r has its minimum value of a minus ae, that's a times one minus-e. As we dial theta up to higher values, r increases, and at theta equals pi, 180°, r achieves its maximum value of a times one plus-e. Then it shrinks back down as theta wraps around to 2pi.

To find the equation, we start with the fact that at any point on the ellipsis, the sum of distances to the foci is equal to the major-axis length, 2a. We can write that as r plus r-prime equals 2a, where r is the distance to the focus at the origin, and r-prime is the distance to the other focus. But that's not such a convenient equation. We want it purely in terms of r and theta, not r-prime, so how do we get rid of the r-prime? We use the Law of Cosines.

The Law of Cosines

The Pythagorean theorem says c-squared is equal to a-squared plus b-squared, where a, b, and c are the lengths of the sides of a right triangle. The Law of Cosines is the generalization to any triangle. It says that for any triangle, c-squared equals a-squared plus b-squared minus 2ab times the cosine of gamma, where gamma is the angle across from the c-side.

We'll apply it to our triangle, with r-prime as our c-side. So, we have r-prime squared equals r-squared plus 2ae -squared minus 2r times 2ae times the cosine of angle opposite r-prime, which is pi minus theta. And the cosine of pi minus theta is minus the cosine of theta. At the end, we find r equals a times one minus e-squared divided by one plus e cos theta. That's our equation!

Semi Major Axis and Eccentricity

So, does it make sense? When the eccentricity is zero, the equation reduces to r equals a times one over one, that's just r equals a; that's a circle. And when e is not zero, and we dial theta around the clock, cos theta goes from one to minus one and back to one, the denominator starts big and gets small, then big again. That, too, makes sense; it says r oscillates between a minimum value at theta equals zero and a maximum at pi.

In particular, if we plug in theta equals 0, we get a times one minus e-squared over one plus e. And since one minus e-squared is one plus e times one minus e, the one plus e's cancel out, and we're left with a times one minus-e, which is what we expected. That's the minimum distance from the focus. Likewise, when theta equals pi, we get the expected maximum distance of a times one plus-e.

So, what Kepler noticed, his 'first law', is that all the planets move on ellipses, with the Sun not at the center but rather at one of the foci. Each planet has its own value for the semi major axis and eccentricity.

Application: C

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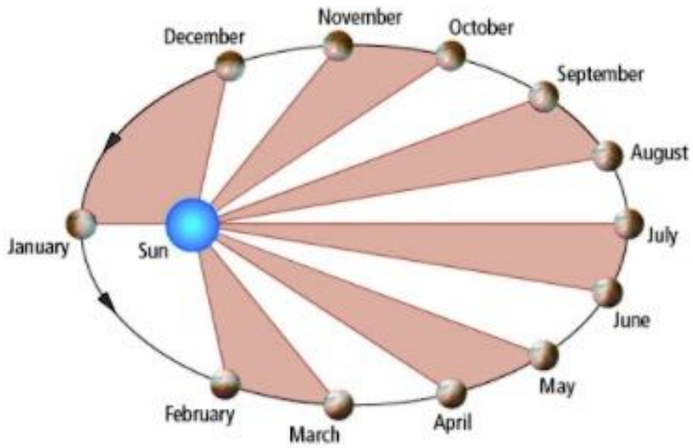
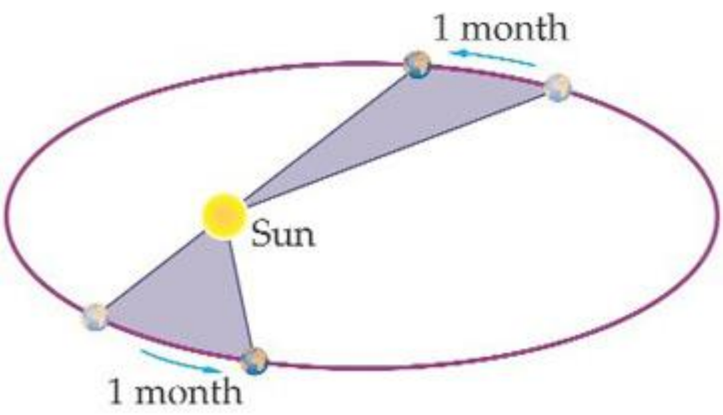
Kepler's Second Law states that a planet sweeps out equal areas in equal time. Using the diagram below, select the correct rank in speed of the Earth (seasons) as it orbits the Sun.



- ○ summer < fall = spring < winter
- ○ summer < fall < spring < winter
- ○ winter < fall = spring < summer
- ○ summer < spring < fall < winter

Conclusion: Kepler's laws of planetary motion govern the orbits of planets around the Sun. At first, Kepler expected the planets to move around the Sun in perfect circles, but after years of observation he found that this was not true. **Kepler's first law** of planetary motion states that the path of each planet around the Sun is an ellipse with the Sun at one focus. This is illustrated by the picture in the section above. Kepler also found that the planets do not move around the Sun at a uniform speed, but move faster when they are closer to the Sun and slower when they are farther away. **Kepler's second law** states that the line from the Sun to any planet sweeps out equal areas of space in equal time intervals. This is shown in the picture below.



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After ten years of work, Kepler discovered the relationship between the time it takes a planet to orbit the Sun and its distance from the Sun. **Kepler's third law** says that the square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet from the Sun. Mathematically, this is given by the ratio T^2/r^3 and applies to all planets. The practical application of Kepler's third law is to calculate the radius of a planet's orbit by observation of that planet's orbital period.

References:

1.Godefroy Wendelin wrote a letter to Giovanni Battista Riccioli about the relationship between the distances of the Jovian moons from Jupiter and the periods of their orbits, showing that the periods and distances conformed to Kepler's third law. See: Joanne Baptista Riccioli, Almagestum novum ... (Bologna (Bononia), (Italy): Victor Benati, 1651), volume 1, page 492 Scholia III. In the margin beside the relevant paragraph is printed: Vendelini ingeniosa speculatio circa motus & intervalla satellitum Jovis. (Wendelin's clever speculation about the movement and distances of Jupiter's satellites.) From p. 492: "III. Non minus Kepleriana ingeniosa est Vendelini ... & D. 7. 164/1000. pro penextimo, & D. 16. 756/1000. pro extimo." (No less clever [than] Kepler's is the most keen astronomer Wendelin's investigation of the proportion of the periods and distances of Jupiter's satellites, which he had communicated to me with great generosity [in] a very long and very learned letter. So, just as in [the case of] the larger planets, the planets' mean distances from the Sun are respectively in the 3/2 ratio of their periods ; so the distances of these minor planets of Jupiter from Jupiter (which are 3, 5, 8, and 14) are respectively in the 3/2 ratio of [their] periods (which are 1.769 days for the innermost [Io], 3.554 days for the next to the innermost [Europa], 7.164 days for the next to the outermost [Ganymede], and 16.756 days for the outermost [Callisto]).)

2. Jump up to:[a] [b] Wilson, Curtis (May 1994). "Kepler's Laws, So-Called" (PDF). HAD News (31): 1– Retrieved December 27, 2016.

3.De la Lande, Astronomie, vol. 1 (Paris, France: Desaint & Saillant, 1764). See, for example:

From page 390: " ... mais suivant la fameuse loi de Kepler, qui sera expliquée dans le Livre suivant (892), le rapport des temps périodiques est toujours plus grand que celui des distances, une planete cinq fois plus éloignée du soleil, emploie à faire sa révolution douze fois plus de temps ou environ; ... " (... but according to the famous law of Kepler, which will be explained in the following book [i.e., chapter] (paragraph 892), the ratio of the periods is always greater than that of the distances [so that, for example,] a planet five times farther from the sun, requires about twelve times or so more time to make its revolution [around the sun] ...)

From page 429: "Les Quarrés des Temps périodiques sont comme les Cubes des Distances. 892. La plus fameuse loi du mouvement des planetes découverte par Kepler, est celle du repport qu'il y a entre les grandeurs de leurs orbites, & le temps qu'elles emploient à les parcourir; ... " (The squares of the periods are as the cubes of the distances. 892. The most famous law of the movement of the planets discovered by Kepler is that of the relation between the sizes of their orbits and the times that the [planets] require to traverse them; ...)

- From page 430: "Les Aires sont proportionnelles au Temps. 895. Cette loi générale du mouvement des planetes devenue si importante dans l'Astronomie, sçavior, que les aires sont proportionnelles au temps, est encore une des découvertes de Kepler; ... " (Areas are proportional to times. 895. This general law of the movement of the planets [which has] become so important in astronomy, namely, that areas are proportional to times, is one of Kepler's discoveries; ...)
- From page 435: "On a appellé cette loi des aires proportionnelles aux temps, Loi de Kepler, aussi bien que celle de l'article 892, du nome de ce célebre Inventeur; ... " (One called this law of areas proportional to times (the law of Kepler) as well as that of paragraph 892, by the name of that celebrated inventor; ...)

4Robert Small, An account of the astronomical discoveries of Kepler (London, England: J Mawman, 1804), pp. 298–299.

5 Robert Small, An account of the astronomical discoveries of Kepler (London, England: J. Mawman, 1804).

6 Bruce Stephenson (1994). Kepler's Physical Astronomy. Princeton University Press. p. 170. ISBN 978-0-691-03652-6.

7Astronomia nova Aitiologitis, seu Physica Coelestis tradita Commentariis de Motibus stellae Martis ex observationibus G.V. Tychnonis.Prague 1609; Engl. tr. W.H. Donahue, Cambridge 1992.

8Johann Planck, 1619), book 5, chapter 3, p. 189. From the bottom of p. 189: "Sed res est certissima exactissimaque quod proportio qua est inter binorum quorumcunque Planetarum tempora periodica, sit

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præcise sesquialtera proportionis mediarum distantiarum, ... " (But it is absolutely certain and exact that the proportion between the periodic times of any two planets is precisely the sesquialternate proportion [i.e., the ratio of 3:2] of their mean distances, ... ") An English translation of Kepler's Harmonices Mundi is available as: Johannes Kepler with E. J. Aiton, A. M. Duncan, and J. V. Field, trans., The Harmony of the World (Philadelphia, Pennsylvania: American Philosophical Society, 1997); see especially p. 411.

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