Volume-09 Issue 02, February-2025 ISSN: 2456-9348
Impact Factor: 8.232

iJETRM
International Journal of Engineering Technology Research & Management
Published By:
https://www.ijetrm.com/

FIBONACCI LABELING IN THE FRAMEWORK OF STAR RELATED GRAPH

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ABSTRACT
The **Fibonacci sequence** is a series of numbers where each number is the sum of the two preceding ones, usually starting with **0 and 1**. Mathematically, it is defined as: F_0=0, F_1=1 F_n= F_{n-1}+F_{n-2}, for $n \geq 2$. This sequence begins as:**0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ….** In this paper, we investigate the existence of some Fibonacci labelings for extended triplicate graph of star.

keywords:
Graph labeling, Triplicate graph, Fibonacci labeling.

1. INTRODUCTION

In 1967, the concept of graph labeling was introduced by Rosa[6] . A graph labeling is an assignment of integers to the edges or vertices or both under certain conditions. In 2011[1], the concept of the extended triplicate graph of a path P_p was introduced by Bala and Thirusangu. In 2023[2], the concept of Extended triplicate graph of star ETG($k_{1,p}$)was introduced by Bala.et.al.,. In 2017 [5], Rokad.et.al., introduced the conceptualization of **Fibonacci cordial labeling**. Let G be a simple graph with p vertices and q edges. An injective function S: δ (G) → $\{F_0, F_1, F_2, F_3, \ldots, F_p\}$, Where F_j is the jth Fibonacci number is said to be Fibonacci cordial labeling, if an induced function S*:β (G) → {0,1} defined by S*(bc) = (S(b)+ S(c))(mod2), ∀ $bc \in \beta(G)$ satisfies the condition $|\beta_{S^*}(1) - \beta_{S^*}(0)| \leq 1$. A graph which admits a Fibonacci cordial labeling is called as Fibonacci cordial graph.

In 2019 [7], Tessymol Abraham.et.al., introduced the concept of **Fibonacci product cordial labeling**. Let G be a simple graph with p vertices and q edges. An injective function S: δ (G) → $\{F_1, F_2, F_3, \ldots, F_p\}$, Where F_j is the jth Fibonacci number is said to be Fibonacci product cordial labeling, if an induced function S*:β(G) → {0,1} defined by S*(bc) = (S(b)S(c))(mod2), ∀ $bc \in \beta(G)$ satisfies the condition $|\beta_{S^*}(1) - \beta_{S^*}(0)| \leq 1$. A graph which admits a Fibonacci product cordial labeling is called as Fibonacci product cordial graph. In 2018 [3], Sekar et.al., introduced the concept of **Fibonacci prime labeling**. Let G be a simple graph with p vertices and q edges. An injective function S: δ (G) → $\{F_2, F_3, \ldots, F_{p+1}\}$, Where F_j is the jth Fibonacci number is said to be Fibonacci prime labeling, if an induced function S*:β (G) → N defined by S*(bc) = $g.c.d(S(b), S(c)) = 1$,∀$bc \in \beta$(G) . A graph which admits a Fibonacci prime labeling called as Fibonacci prime graph.

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In 2018 [8], Thirugnanasambandam.et.al., oriented the concept of **Fibonacci antimagic labeling**. A bijective function S: δ (G) → $\{F_0, F_1, F_2, F_3, \ldots, F_p\}$, Where F_j is the j[th] Fibonacci number is said to be Fibonacci antimagic labeling, if an induced function S*:β (G) →{1,2,…,2p} is defined by S*(bc) = (S(b)+S(c)) ,$\forall bc \in \beta$(G) . Then the resulting edge labelings are distinct. A graph which admits a Fibonacci antimagic labeling is called as Fibonacci antimagic graph.
Inspired by the above studies, In this paper we investigate the existence of Fibonacci cordial labeling, Fibonacci product cordial labeling, Fibonacci divisor cordial labeling, Fibonacci prime labeling and Fibonacci antimagic labeling in the context of Extended Triplicate of star graph.

2. MAIN RESULT

In this section, we investigate the existence of Fibonacci cordial labeling, Fibonacci product cordial labeling, Fibonacci divisor cordial labeling, Fibonacci prime labeling, Fibonacci antimagic labeling for the Extended Triplicate of star graph.

2.1 STRUCTURE OF EXTENDED TRIPLICATE OF STAR GRAPH

Let G be a star graph ($K_{1,p}$).The triplicate graph of star with vertex set δ'(G) and edge set β'(G) is given by: δ'(G) = $\{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \le i \le p\}$ and β'(G) = $\{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i'/1 \le i \le p\}$. Clearly, Triplicate graph of star TG($K_{1,p}$) with this vertex set and edge set is disconnected. To make this a connected graph, include a new edge bc_1 to the edge set β'(G). Thus, we get an **Extended Triplicate of star graph** with vertex *set* $\delta = \delta'$ and edge set $\beta(G) = \beta'(G) \cup bc_1$. Clearly, ETG($K_{1,p}$) has $3(p+1)$ vertices and $(4p+1)$ edges and is denoted by ETG($K_{1,p}$).

Theorem 2.1: Extended triplicate of star graph is a Fibonacci cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has vertex set δ(G) $= \{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \le i \le p\}$ and edge set βG) = $\{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i' \cup bc_1/1 \le i \le p\}$
Clearly, it has $3(p+1)$ vertices and $(4p+1)$ edges.
To show that ETG($K_{1,p}$) is a Fibonacci cordial graph.
Define an injective function $S: \delta$ (G) → $\{F_0, F_1, F_2, F_3, \ldots, F_{3(p+1)}\}$ to label the vertices as follows.

<table>
<tr><td rowspan="4">$if, p = 3j;$
$j \in$ N</td><td>$S(b) = F_2$</td><td>$S(b') = F_0$</td><td>$S(b'') = F_4$</td></tr>
<tr><td>For, $1 \le i \le p$</td><td>$S(c_i) = F_{2i-1}$</td><td>$S(c_i') = F_{2i+4}$</td></tr>
<tr><td colspan="3">To find $S(c_i'')$</td></tr>
<tr><td>$p \equiv 1\ mod2$</td><td colspan="2">$S(c_i'') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+3}{2} \\ F_{2i+p-1} & ; \frac{p+5}{2} \le i \le p \end{cases}$</td></tr>
</table>

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<table>
<tr><td></td><td>$p \equiv 0\ mod2$</td><td colspan="2">$S(c_i'') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+2}{2} \\ F_{2(i+1)+p} & ; \frac{p+4}{2} \le i \le p \end{cases}$</td></tr>
<tr><td rowspan="6">$if,$
$p = 3j - 1$;
$j \in$ N</td><td>$S(b) = F_2$</td><td>$S(b') = F_0$</td><td>$S(b'') = F_4$</td></tr>
<tr><td>For, $1 \le i \le p$</td><td>$S(c_i) = F_{2i-1}$</td><td>$S(c_i') = F_{2i+4}$</td></tr>
<tr><td colspan="3">To find $S(c_i'')$</td></tr>
<tr><td>$p \equiv 1\ mod2$</td><td colspan="2">$S(c_i'') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+3}{2} \\ F_{2i+p+1} & ; \frac{p+5}{2} \le i \le p \end{cases}$</td></tr>
<tr><td>$p \equiv 0\ mod2$</td><td colspan="2">$S(c_i'') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+2}{2} \\ F_{2(i+1)+p} & ; \frac{p+4}{2} \le i \le p \end{cases}$</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="7">$if,$
$p = 3j + 1$;
$j \in$ N</td><td>$S(b) = F_2$</td><td>$S(b') = F_3$</td><td>$S(b'') = F_1$</td></tr>
<tr><td>$S(c_1) = F_{3(p+1)}$</td><td colspan="2">$S(c_2) = F_4$</td></tr>
<tr><td>For, $3 \le i \le p$</td><td colspan="2">$S(c_i) = F_{2i-1}$</td></tr>
<tr><td>For, $1 \le i \le p$</td><td colspan="2">$S(c_i'') = F_{2i+4}$</td></tr>
<tr><td colspan="3">To find $S(c_i')$</td></tr>
<tr><td>$p \equiv 1\ mod2$</td><td colspan="2">$S(c_i') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+3}{2} \\ F_{2i+p-1} & ; \frac{p+5}{2} \le i \le p \end{cases}$</td></tr>
<tr><td>$p \equiv 0\ mod2$</td><td colspan="2">$S(c_i') = \begin{cases} F_{2(p+i)-1} & ; 1 \le i \le \frac{p+2}{2} \\ F_{2(i+1)+p} & ; \frac{p+4}{2} \le i \le p \end{cases}$</td></tr>
</table>

Define an induced function $S^*:\beta(G) \rightarrow \{0,1\}$ by $S^*(bc) = (S(b) + S(c))(mod2), \forall\ bc\ \epsilon\ \beta(G)$ to get the edge labels as follows:

$if, p = 3j;\ j \in$ N	$\beta_{S^*}(0) = 2p + 1$	$\beta_{S^*}(1) = 2p$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p + 1 - 2p\rvert \le 1$

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$if, p = 3j - 1;$ $j \in \mathrm{N}$	$\beta_{S^*}(0) = 2p$	$\beta_{S^*}(1) = 2p + 1$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p - (2p + 1)\rvert \leq 1$
$if, p = 3j + 1;$ $j \in \mathrm{N}$	$\beta_{S^*}(0) = 2p + 1$	$\beta_{S^*}(1) = 2p$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert(2p + 1) - 2p\rvert \leq 1$

From the above three cases, it is clear that the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$ is satisfied.
Hence, Extended Triplicate of star graph is a Fibonacci cordial graph.

EXAMPLE 2.1: ETG($K_{1,3}$), ETG($K_{1,4}$) and ETG($K_{1,5}$) and its Fibonacci cordial labelling is shown in figure 1, figure 2 and figure 3.

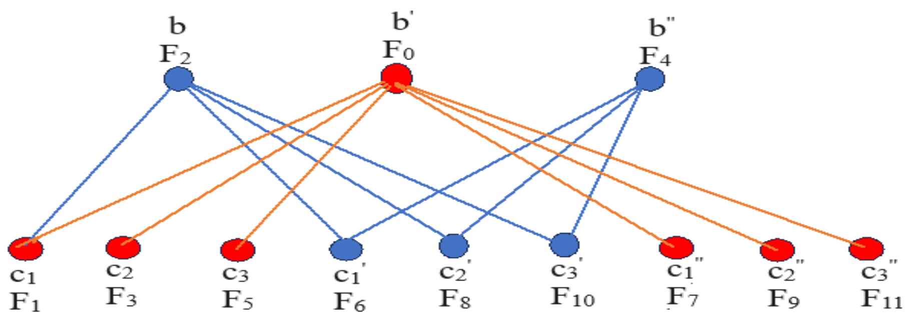

FIGURE – 1

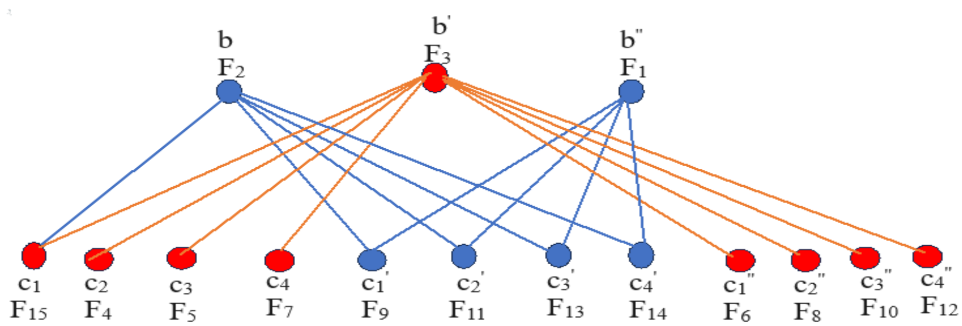

FIGURE – 2

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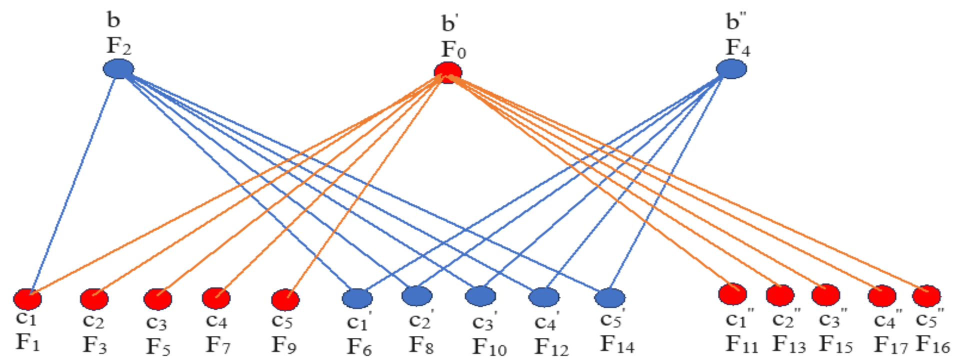

FIGURE – 3

Theorem 2.2: Extended triplicate of star graph is a Fibonacci product cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has vertex set δ(G) $= \{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \leq i \leq$ p } and edge set βG) $= \{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i' \cup$ bc$_1/1 \leq i \leq p\}$

Clearly, it has $3(p+1)$ vertices and $(4p+1)$ edges.

To show that ETG($K_{1,p}$) is a Fibonacci product cordial graph.

Define an injective function S: δ (G) → $\{F_1, F_2, F_3, \ldots, F_{3(p+1)}\}$ to label the vertices as follows.

$S(b) = F_2$	$S(b') = F_1$	$S(b'') = F_3$
For, $1 \leq i \leq p$	$S(c_i) = F_{2(i+1)}$	$S(c_i') = F_{2i+3}$
To find $S(c_i'')$		
$if, p \equiv 1\ mod2$	S(c_i'') $= \begin{cases} F_{2(p+i+1)}; & 1 \leq i \leq \frac{p+1}{2} \\ F_{p+2(i+1)}; & \frac{p+3}{2} \leq i \leq p \end{cases}$	
$if, p \equiv 0\ mod2$	S(c_i'') $= \begin{cases} F_{2(p+i+1)}; & 1 \leq i \leq \frac{p}{2} \\ F_{2i+p+3}; & \frac{p+2}{2} \leq i \leq p \end{cases}$	

Define an induced function S*:β (G) → {0,1} by S*(bc) = $(S(b)\,S(c))(mod2)$, ∀ $bc \in \beta(G)$ to get the edge labels as follows:

$\beta_{S^*}(1) = (2p+1)$ and $\beta_{S^*}(0) = 2p$

Thus, $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |2p - (2p+1)| \leq 1$.

Hence, Extended triplicate of star graph is a Fibonacci product cordial graph.

EXAMPLE 2.2: ETG($K_{1,3}$), ETG($K_{1,4}$) and its Fibonacci product cordial labeling is shown in figure 4, figure 5.

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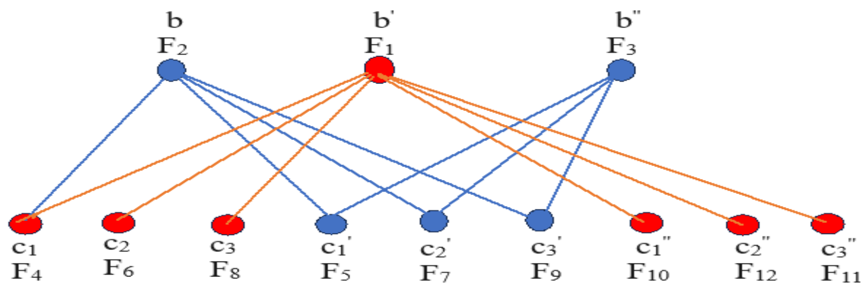

FIGURE – 4

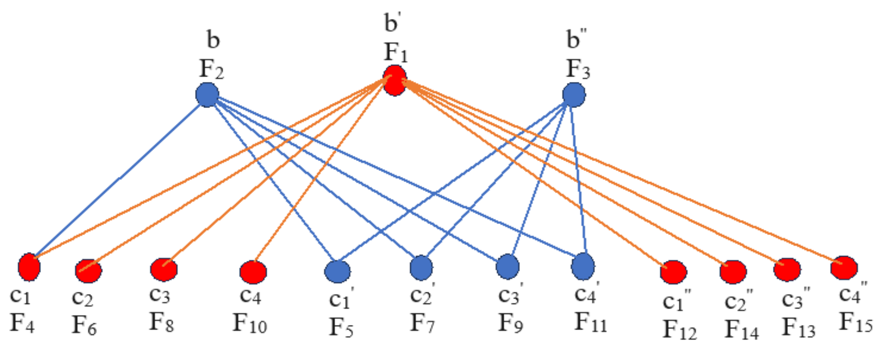

FIGURE – 5

Theorem 2.3: Extended triplicate of star graph is a Fibonacci prime graph.
Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has vertex set
δ(G) = $\{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \ \le \ i \le \ \text{p}\}$ and edge set
βG) = $\{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i' \cup \text{bc}_1/1 \le i \le p\}$
Clearly, it has $3(p+1)$ vertices and $(4p+1)$ edges.
To show that ETG($K_{1,p}$) is a Fibonacci prime graph.
Define an injective function S: δ (G) → $\{F_2, F_3, \ldots, F_{3(p+1)+1}\}$ to label the vertices as follows.

$S(b) = F_3$	$S(b') = F_2$	$S(b'') = F_4$	$S(c_1) = F_5$
For,$2 \le i \le p$	$S(c_i) = F_{3i}$		
For,$1 \le i \le p$	$S(c_i') = F_{4+3i}$		
For,$1 \le i \le p-1$	$S(c_i'') = F_{3i+5}$		$S(c_p'') = F_{3(p+1)}$

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By the vertex labels and using the induced function $S^{*}:\beta$ (G) $\rightarrow$ N defined by S^{*}(b c) = $g.c.d\ (S(b), S(c))\ = 1, \forall bc\ \epsilon\ \beta$(G)
Thus, we get each and every edges are relatively prime.
Hence, Extended triplicate of star graph is a Fibonacci prime graph.
EXAMPLE 2.3: ETG($K_{1,3}$) and its Fibonacci prime labeling is shown in figure 6.

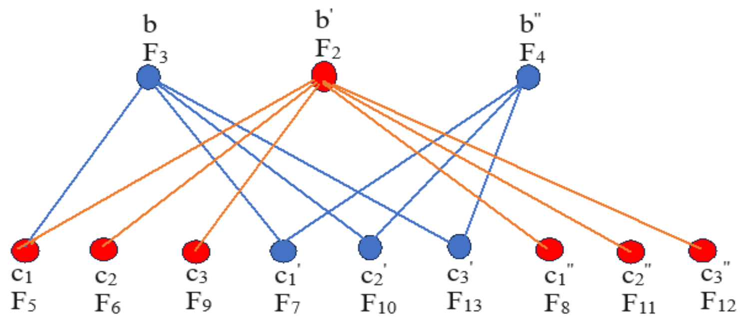

FIGURE – 6

Theorem 2.4: Extended triplicate of star graph is a Fibonacci antimagic graph.
Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has vertex set δ(G) $= \{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1\ \leq\ i \leq\ $ p $\}$ and edge set βG) $= \{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i' \cup$ b$c_1/1 \leq i \leq p\}$. Clearly, it has $3(p + 1)$ vertices and $(4p + 1)$ edges.
To show that ETG($K_{1,p}$) is a Fibonacci anti magic graph.
Define the bijective function S: δ (G) $\rightarrow$ $\{ F_0, F_1, F_2, F_3, \ldots, F_{3(p+1)} \}$ to label the vertices as follows.

$S(b) = F_1$	$S(b') = F_0$	$S(b'') = F_3$
For, $2 \leq i \leq p$	$S(c_i) = F_{i+2}$	$S(c_1) = F_2$
For, $1 \leq i \leq p$	$S(\ c_1') = F_{2(p+1)+i}$	$S(c_i'') = F_{p+i+2}$

Define an induced function $S^{*}:\beta$ (G) $\rightarrow\{1,2,\ldots,2F_{3(p+1)}\}$ by $S^{*}(bc) = (S(b) + S(c))$, ∀ bc ∈ β(G) to obtain the label for edges as follows.

$S^{*}(bc_1) =\ F_1 + F_2$	
For, $1 \leq i \leq p$	
$S^{*}(\ bc_i') =\ F_1\ + F_{2(p+1)+i}$	$S^{*}(b'c_i) = F_0\ + F_{i+2}$
$S^{*}(b'c_i'') =\ F_0\ + F_{p+i+2}$	$S^{*}(b''c_i') = F_3\ \ + F_{2(p+1)+i}$

Thus, the resulting edge labels are distinct.
Hence, Extended triplicate of star graph is a Fibonacci anti magic graph.
EXAMPLE 2.5: ETG($K_{1,3}$) and its Fibonacci anti magic labeling is shown in figure 7.

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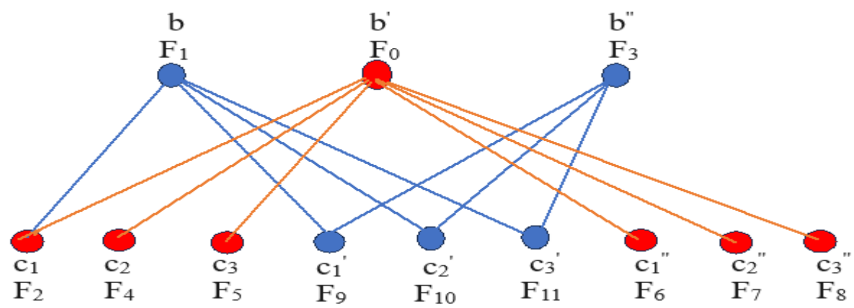

FIGURE – 7

CONCLUSION

In this paper, we have proved that extended triplicate of star graph admits Fibonacci cordial labeling, Fibonacci product cordial labeling, Fibonacci prime labeling, Fibonacci antimagic labeling.

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