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CORDIAL LABELING IN THE CONTEXT OF TWIG RELATED GRAPH

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ABSTRACT
In this paper we investigate the existence of Cordial Labeling for Twig related graph.

Keywords:
Graph Labeling, Cordial Labeling, Tribonacci Labeling, Comb Graph

1. INTRODUCTION

The concept of graph Labeling was introduced by Rosa in 1967 [5]. Bala et.al.,2020 [1] discussed the concept of Cordial Labeling for Some Special Kite Graph.Murali.et.al., 2015 [3] introduced the concept of Biconditional Cordial Labeling. Sundaram et.al., 2005 [7] introduced the concept of Prime Cordial Labeling. Sarbari Mitra et.al.,2022 [6] introduced the concept of Tribonacci Cordial Labeling. Nellai Murugan et. Al., 2015 [4] discussed the concept of Twig Graph, Bala et.al., 2019 [2] discussed the concept of Comb graph. Motivated by the above work, in this paper we have investigate and the existence of Cordial Labeling, Biconditional Cordial Labeling, Prime Cordial Labeling and Tribonacci Cordial Labeling for Twig$(P_m \odot K_1)$, $m \geq 2$.

2. PRELIMINARIES

In this section, we provide some basic definitions relevant to this paper.

Definition 2.1: A graph G is said to admit a cordial labeling if there exists a function $h : \zeta(G) \rightarrow \{0,1\}$ such that the induced function $h^*: \tau(G) \rightarrow \{0,1\}$ defined as $h^*(\zeta_i\zeta_j) = |\left(h(\zeta_i) - h(\zeta_j)\right)|$ or $\left(h(\zeta_i) + h(\zeta_j)\right)(\text{mod}2)$ satisfies the property that the number of vertices labeled '0' and the number of vertices labeled '1' differ by at most one and the number of edges labeled '0' and the number of edges labeled '1' differ by at most one.

Definition 2.2: Let $G(\zeta,\tau)$ be a graph.A function $h : \zeta(G) \rightarrow \{0,1\}$ of a graph G is said to be Biconditional Cordial labeling if there exists an induced edge function $h^*:\tau(G) \rightarrow \{0,1\}$ defined by $h(\zeta_i\zeta_j) = \begin{cases} 1, & \text{if } h(\zeta_i) = h(\zeta_j) \\ 0, & \text{if } h(\zeta_i) \neq h(\zeta_j) \end{cases}$ for all $\zeta_i, \zeta_j \in \tau$, which satisfices the conditions.$|\zeta_h(0) - \zeta_h(1)| \leq 1$ and $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$.

Definition 2.3: A Prime Cordial labeling of a graph G with the vertex set ζ (G) is bijection $h: \zeta(G) \rightarrow \{1,2,..|\zeta(G)|\}$ such that each edge uv is assigned the label '1' if $\gcd(h(\zeta_i), h(\zeta_j)) = 1$ and '0' if $\gcd(h(\zeta_i), f(\zeta_j)) > 1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost '1'. A graph which admits Prime Cordial labeling is called Prime Cordial graph.

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Definition 2.4: The sequence T_n of Tribonacci number is defined by the third order linear recurrence relation (for n≥ 0):

$$T_{n+3} = T_n + T_{n+1} + T_{n+2}; T_0 = 0, T_1 = T_2 = 1$$

An injective function $h: \zeta(G) \to \{T_0, T_1, \ldots, T_n\}$ is said to be Tribonacci cordial labeling if the induced function $h^*: \tau(G) \to \{0,1\}$ defined by

$$h^*(\zeta_i\zeta_j) = (h(\zeta_i) + h(\zeta_j))(\mathrm{mod}2)$$

satisfies the condition $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$. A graph which admits Tribonacci cordial labeling is called Tribonacci cordial graph.

Definition 2.5: Let P_m be a path graph with m vertices. The comb graph is defined as $P_m \odot K_1$. It has 2m vertices and 2m-1 edges.

Definition 2.6: A graph obtained from a path by attaching exactly two pendent edges to each internal vertex of the path is called a twig and is denoted by $Tg_m, m \geq 1$.

Definition 2.7: Twig($P_m \odot K_1$), $m \geq 2$ is obtained by attaching exactly two pendent edges to each internal vertex of the external path of comb graph. The vertex set and edge set are defined as follows $\zeta(G) = \{\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_{4m}\}$ and $\tau(G) = \left\{\{\zeta_i\zeta_{i+1}/1 \leq i \leq m-1\} \cup \{\{\zeta_i\zeta_{m+i}\} \cup \{\zeta_i\zeta_{2m+(2i-1)}\} \cup \{\zeta_i\zeta_{2m+2i}\}/1 \leq i \leq m\}\right\}$.

Clearly, Twig($P_m \odot K_1$), $m \geq 2$ has 4m vertices and $4m - 1$ edges.The graph thus obtained is a particular case of a uniform caterpillar.

3. MAIN RESULT

In this section, we discuss about the structure of the Twig($P_m \odot K_1$), $m \geq 2$for Cordial labeling, Biconditional Cordial labeling, Prime Cordial labeling and Tribonacci Cordial labeling.

THEOREM 3.1:

Twig($P_m \odot K_1$), $m \geq 2$ admits Cordial labeling.

Proof:

From the structure of Twig($P_m \odot K_1$), $m \geq 2$. It is clear that Twig($P_m \odot K_1$), $m \geq 2$ has 4m vertices and $4m - 1$ edges.

Define the function $h: \zeta(G) \to \{0,1\}$ to obtain the vertex lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i = \zeta_{2m+(2i-1)} = 1$

(ii) $\zeta_{i+m} = \zeta_{2m+2i} = 0$

Clearly, $\zeta_h(0) = 2m$, $\zeta_h(1) = 2m$,

Therefore, $|\zeta_h(0) - \zeta_h(1)| = |2m - 2m| = 0 \leq 1$

Define the function $h^*: \tau(G) \to \{0,1\}$ to obtain edge lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i\zeta_{i+1} = \zeta_i\zeta_{2m+(2i-1)} = 0$

(ii) $\zeta_i\zeta_{i+m} = \zeta_i\zeta_{2m+2i} = 1$

Clearly, $\tau_{h^*}(0) = 2m - 1$, $\tau_{h^*}(1) = 2m$,

Therefore, $|\tau_{h^*}(0) - \tau_{h^*}(1)| = |(2m - 1) - 2m| = 1 \leq 1$

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Hence the conditions $|\zeta_h(0) - \zeta_h(1)| \leq 1$ and $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$ are satisfied.
Therefore, $Twig(P_m \odot K_1)$, $m \geq 2$ admits cordial labeling.

EXAMPLE 3.1:

Cordial labeling for $Twig(P_4 \odot K_1)$ is shown in the figure.3.1

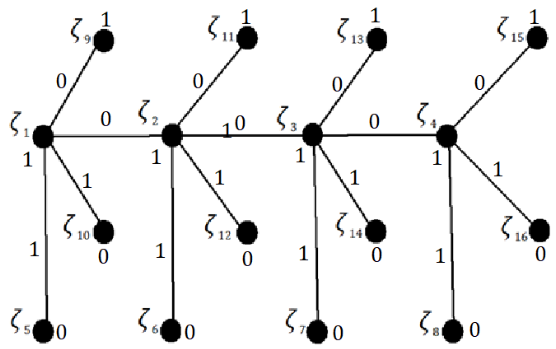

Figure 3.1

THEOREM 3.2:

$Twig(P_m \odot K_1)$, $m \geq 2$ admits Biconditional Cordial labeling.

Proof:

From the structure of $Twig(P_m \odot K_1)$, $m \geq 2$. It is clear that $Twig(P_m \odot K_1)$, $m \geq 2$ has $4m$ vertices and $4m - 1$ edges.
Define the function $h : \zeta(G) \to \{0,1\}$ to obtain the vertex lables as follows:
For $1 \leq i \leq m$

(i) $\zeta_i = \zeta_{2m+(2i-1)} = 1$

(ii) $\zeta_{i+m} = \zeta_{2m+2i} = 0$

Clearly, $\zeta_h(0) = 2m$, $\zeta_h(1) = 2m$,
Therefore, $|\zeta_h(0) - \zeta_h(1)| = |2m - 2m| = 0 \leq 1$
Define the function $h^*: \tau(G) \to \{0,1\}$ to obtain edge lables as follows:
For $1 \leq i \leq m$

(i) $\zeta_i\zeta_{i+1} = \zeta_i\zeta_{2m+(2i-1)} = 1$

(ii) $\zeta_i\zeta_{i+m} = \zeta_i\zeta_{2m+2i} = 0$

Clearly, $\tau_{h^*}(0) = 2m$, $\tau_{h^*}(1) = 2m - 1$,
Therefore, $|\tau_{h^*}(0) - \tau_{h^*}(1)| = |2m - (2m - 1)| = 1 \leq 1$
Hence the conditions $|\zeta_h(0) - \zeta_h(1)| \leq 1$ and $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$ are satisfied.
Therefore, $Twig(P_m \odot K_1)$, $m \geq 2$ admits Biconditional cordial labeling.

EXAMPLE 3.2:

Biconditional Cordial labeling for $Twig(P_4 \odot K_1)$ is shown in the figure.3.2

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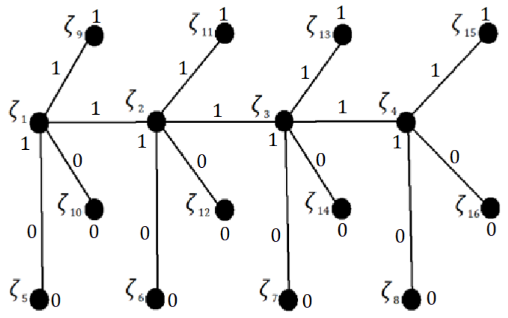

figure.3.2

THEOREM 3.3:

$Twig(P_m \odot K_1)$, $m \geq 2$ admits Prime Cordial labeling.

Proof:

From the structure of $Twig(P_m \odot K_1)$, $m \geq 2$. It is clear that $Twig(P_m \odot K_1)$, $m \geq 2$ has $4m$ vertices and $4m - 1$ edges.

Define the function $h : \zeta(G) \rightarrow \{1,2,..,|\zeta(G)|\}$ to obtain the vertex lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i = 4i - 2$

(ii) $\zeta_{i+m} = 4i$

(iii) $\zeta_{2m+(2i-1)} = 4i - 3$

(iv) $\zeta_{2m+2i} = 4i - 1$

Define the function $h^* : \tau(G) \rightarrow \{0,1\}$ to obtain edge lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i\zeta_{i+1} = \zeta_i\zeta_{i+m} = 0$

(ii) $\zeta_i\zeta_{2m+(2i-1)} = \zeta_i\zeta_{2m+2i} = 1$

Clearly, $\tau_{h^*}(0) = 2m - 1$, $\tau_{h^*}(1) = 2m$,

Therefore, $|\tau_{h^*}(0) - \tau_{h^*}(1)| = |(2m - 1) - 2m| = 1 \leq 1$

Hence the condition $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$ are satisfied.

Therefore, $Twig(P_m \odot K_1)$, $m \geq 2$ admits Prime cordial labeling.

EXAMPLE 3.3

Prime Cordial labeling for $Twig(P_4 \odot K_1)$ is shown in the figure.3.3

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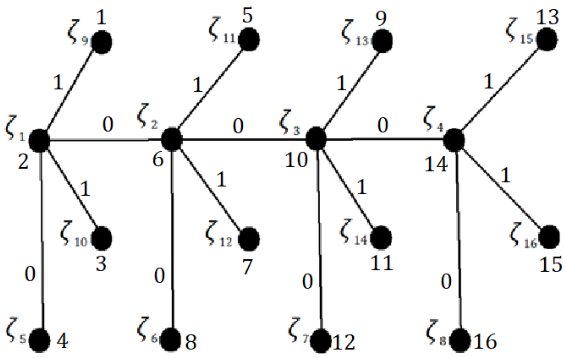

figure.3.3

THEOREM 3.4:

Twig($P_m \odot K_1$), $m \geq 2$ admits Tribonacci Cordial labeling.

Proof:

From the structure of Twig($P_m \odot K_1$), $m \geq 2$. It is clear that Twig($P_m \odot K_1$), $m \geq 2$ has $4m$ vertices and $4m - 1$ edges.

Define the function $h : \zeta(G) \rightarrow \{T_0, T_1, \dots, T_n\}$ to obtain the vertex lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i = T_{4i-1}$

(ii) $\zeta_{i+m} = T_{4i}$

(iii) $\zeta_{2m+(2i-1)} = T_{4i-3}$

(iv) $\zeta_{2m+2i} = T_{4i-2}$

Define the function $h^* : \tau(G) \rightarrow \{0,1\}$ to obtain edge lables as follows:

For $1 \leq i \leq m$

(i) $\zeta_i\zeta_{i+1} = \zeta_i\zeta_{i+m} = 0$

(ii) $\zeta_i\zeta_{2m+(2i-1)} = \zeta_i\zeta_{2m+2i} = 1$

Clearly, $\tau_{h^*}(0) = 2m - 1$, $\tau_{h^*}(1) = 2m$,

Therefore, $|\tau_{h^*}(0) - \tau_{h^*}(1)| = |(2m - 1) - 2m| = 1 \leq 1$

Hence, the condition $|\tau_{h^*}(0) - \tau_{h^*}(1)| \leq 1$ are satisfied.

Therefore, Twig($P_m \odot K_1$), $m \geq 2$ admits Tribonacci cordial labeling.

EXAMPLE 3.4:

Tribonacci Cordial labeling for Twig($P_4 \odot K_1$) is shown in the figure.3.4.

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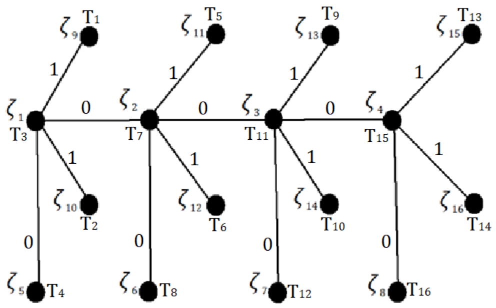

figure.3.4

CONCLUSION:

In this paper, we have proved the existence of Cordial Labeling, Biconditional Cordial Labeling, Prime Cordial Labeling and Tribonacci Labeling for Twig($P_m \odot K_1$), $m \geq 2$.

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