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SPECIAL CLASS OF GRAPH WITH SOME LABELING

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ABSTRACT
In this paper we investigate the existence of some graph labelings in Twig($P_m \odot K_1$), $m \geq 2$.

Keywords:
Graph Labeling, Tribonacci numbers, Gaussian Labeling, Lucky Labeling.

1. INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967 [7]. In 2019 Esakkiammal et.al., introduced the concept of Proper d-lucky labeling [5], Proper Lucky labeling was introduced by Kins Yenoke et.al.,(2016) [1], In 2004 Sundaram et.al., introduced the concept of Product Cordial Labeling (2022) [9], Tribonacci cordial labeling[was introduced by Sarbari Mitra et.al., 2022 [8],In 2024 Bala et.al., introduced the concept of Tribonacci Product cordial labeling[2]. Gaussian anti magic labeling was introduced by Thirusangu et.al., (2019) [10]. Motivated by the above work, in this paper we have investigate and the existence of d-lucky Labeling, Luck Labeling, Tribonacci Product cordial labeling for Twig($P_m \odot K_1$), $m \geq 2$.

2. PRELIMINARIES

In this section, we provide some basic definitions relevant to this paper.

Definition 2.1: Let h be a function from R (G) to {0,1}. For each edge $r_i r_j$, assign the label h(r_i)h(r_j). H is called product cordial labeling if $|r_h(0) - r_h(1)| \leq 1$ and $|b_{h^*}(0) - b_{h^*}(1)| \leq 1$ where $r_h(i)$ and $b_{h^*}(i)$ denote the number of vertices and edges respectively label in with '0' and '1' .A graph with a product cordial abeling is called a product cordial graph.

Definition 2.2: A d-lucky labeling is called proper if $l(u) \neq l(r)$ for every adjacent vertices u and r. The proper d-lucky number of a graph is the least positive integ k such that G has a Proper d-lucky labeling with $\{1,2,\ldots.,k\}$ as the set of labels and is denoted by $\eta_{pdl}(G)$.

Definition 2.3: A Lucky Labeling is Proper lucky abeling if the labeling l is proper as well as lucky, that is if u and r are adjacent in G then $h(u) \neq h(r)$ and if $s(u) \neq s(r)$. The Proper Lucky labeling with $\{1,2,\ldots.,k\}$ the set of labels.

Definition 2.4: The sequence $\{T_m\}_{m=1}^{\infty}$ of Tribonacci numbers is defined by the third order linear recurrence relation (for m > 0):

$$T_{m+3} = T_m + T_{m+1} + T_{m+2}\,;\; T_0 = 0, T_1 = T_2 = 1$$

$$\{T_m\} = [1,1,2,4,7\ 13,24,..]$$

Definition 2.5: An injective function $\delta : R(G) \rightarrow \{T_0, T_1, \ldots T_m\}$ is said to be Tribonacci cordial labeling if the induced function δ^*: B(G)→ $\{0,1\}$ defined by $\delta^*(r_i r_j) = (\delta(r_i) + \delta(r_j))(mod 2)$ satisfies the condition $|b_{\delta^*}(0) - b_{\delta^*}(1)| \leq 1$. A graph which admits Tribonaci cordial labeling is called Tribonacci cordial graph.

Definition 2.6: An injective function $\delta : R(G) \rightarrow \{T_1, T_2, \ldots T_m\}$ is said to be Tribonacci product cordial labeling if the induced function δ^*: B(G)→ $\{0,1\}$ defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod 2)$ satisfies the condition

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$| b_{\delta^*}(0) - b_{\delta^*}(1)| \leq 1$. A graph which admits Tribonacci Product cordial labeling is called Tribonacci product cordial graph.

Definition 2.7: Gaussian antimagic labeling in a G(R,B) graph is a function $h: R(G) \rightarrow \{c + id \ / \ c, d \in \ N \ \} 1 \leq c \leq d \leq b$ such that the induced function $h^* : B(G) \rightarrow N \ such \ that \ \ h^*(ru) = |h(r)|^2 + |h(u)|^2$ results all the edge labels are distinct.A graph which admits Gaussian antimagic labeling is called Gaussian antimagic graph.

Definition 2.8: Let P_m be a path graph with m vertices. The comb graph is defined as $P_m \odot K_1$. It has 2m vertices and 2m-1 edges.

Note: A comb is a caterpillar in which each vertex in the path is with K_1.

Definition 2.9: A graph obtained from a path by attaching exactly two pendent edges to each internal vertex of the path is called a twig and is denoted by $(Twig)_m, m \geq 1$.

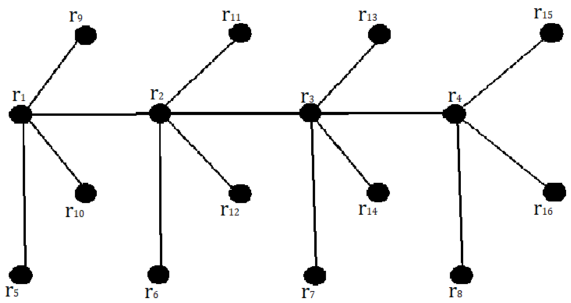
3. MAIN RESULT

In this section, we discuss about the structure of the Twig($P_m \odot K_1$) graph for Proper d-lucky labeling, Proper Lucky labeling, Tribonacci Product cordial labeling and Gaussian antimagic labeling.

Structure of Twig($P_m \odot K_1$) :

Twig($P_m \odot K_1$), $m \geq 2$ is a graph obtained by attaching exactly two pendent edges to each internal vertex of the external path of comb graph. The vertex set and edge set are defined as follows $R(G) = \{r_1, r_2, r_3, .., r_{4m}\}$ and $B(G) = \left\{\{r_i r_{i+1} / 1 \leq i \leq m-1\} \cup \{\{r_i r_{m+i}\} \cup \left\{r_i r_{2m+(2i-1)}\right\} \cup \{r_i r_{2m+2i}\} / 1 \leq i \leq m\}\right\}$.

Clearly, Twig($P_m \odot K_1$), $m \geq 2$ has $4m$ vertices and $4m - 1$ edges. The graph thus obtained is a particular case of a uniform caterpillar.



THEOREM 3.1:

Twig($P_m \odot K_1$), $m \geq 2$ admits Proper lucky labeling with $\eta_{dl}(Twig(P_m \odot K_1)) = 2$.

Proof:

From the structure of Twig($P_m \odot K_1$) . It is clear that Twig($P_m \odot K_1$) has $4m$ vertices and $4m - 1$ edges.

To prove Twig($P_m \odot K_1$), is lucky, define the function $l: R(G) \rightarrow N$ to label the vertices as follows:

Case(i): $m = 1(mod2)$

(i) $l(r_1) = l(r_m) = 1$,

(ii) $2 \leq i \leq m-1$, m≥ 3, $l(r_i) = \{2, i \equiv 0(mod2) \ 1, \quad otherwise$,

(iii) $1 \leq i \leq m$, $l(r_{i+m}) = \{1, i \equiv 0(mod2) \ 2, \quad otherwise$,

$l\left(r_{2m+(2i-1)}\right) = \{1, i \equiv 0(mod2) \ 2, \quad otherwise$, $l(r_{2m+2i}) = \{1, i \equiv 0(mod2) \ 2, \quad otherwise$.

Case(ii): $m = 0(mod2)$

(i) $l(r_1) = 1$, $l(r_m) = 2$,

(ii) $2 \leq i \leq m-1, m \geq 3, l(r_i) = \{2, i \equiv 0(mod2) \ 1, \quad otherwise$,

(iii) $1 \leq i \leq m$, $l(r_{i+m}) = \{1, i \equiv 0(mod2) \ 2, \quad otherwise$,

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$l(r_{2m+(2i-1)}) = \{1, i \equiv mod2\ 2, otherwise\ ,\ \ l(r_{2m+2i}) = \{1, i \equiv mod2\ 2, otherwise\ .$

Case(i): Therefore,

(i) $s(r_1) = s(r_n) = 8,$

(ii) $2 \leq i \leq m-1, m \geq 3, s(r_i) = \{5, i \equiv 0(mod2)\ 10,\ \ otherwise\ ,$

(iii) $1 \leq i \leq m, s(r_{i+m}) = s(r_{2m+(2i-1)}) = s(r_{2m+2i}) = \{2, i \equiv mod2\ 1, otherwise\ ,$

Case(ii): Therefore,

(i) $s(r_1) = 8, s(r_n) = 4,$

(ii) $2 \leq i \leq m-1, m \geq 3, s(r_i) = \{5, i \equiv 0(mod2)\ 10,\ \ otherwise\ ,$

(iii) $1 \leq i \leq m, s(r_{i+m}) = s(r_{2m+(2i-1)}) = s(r_{2m+2i}) = \{2, i \equiv 0(mod2)\ 1,\ \ otherwise\ .$

Clearly, $s(r_i) \neq s(r_{i+m})$, $s(r_i) \neq s(r_{2m+(2i-1)}), s(r_i) \neq s(r_{2m+2i})$, for any two adjacent vertices of Twig($P_m \odot K_1$) . Therefore Twig($P_m \odot K_1$), $m \geq 2$ admits Proper lucky labeling with $\eta_{dl}(Twig(P_m \odot K_1)) = 2$.

EXAMPLE 3.1:

Proper Lucky labeling for Twig($P_m \odot K_1$) $for\ m = 4\ and\ m = 5$ is shown in the figure 3.1.1 and 3.1.2 respectively.

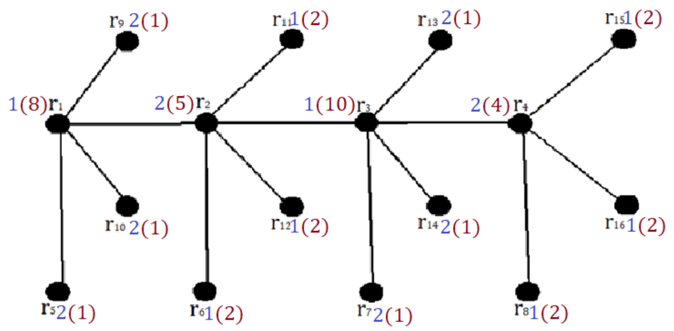

Figure 3.1.1

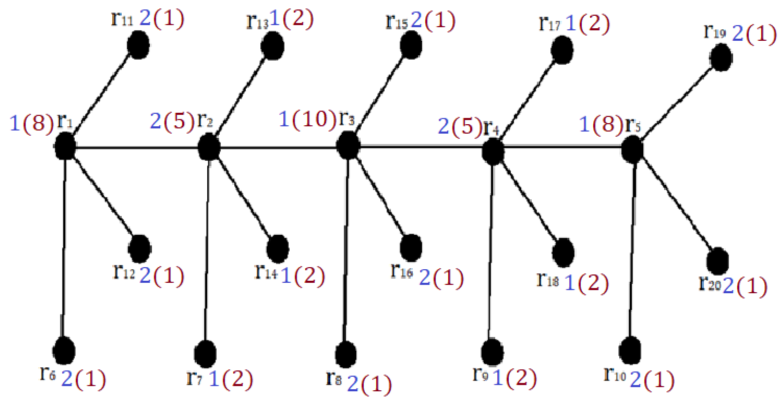

Figure 3.1.2

THEOREM 3.2:

Twig($P_m \odot K_1$), $m \geq 2$ admits Proper d-lucky labeling with $\eta_{dl}(Twig(P_m \odot K_1)) = 2$.

Proof:

From the structure of Twig($P_m \odot K_1$), $m \geq 2$. It is clear that Twig($P_m \odot K_1$), $m \geq 2$ has $4m$ vertices and $4m - 1$ edges.

To prove Twig($P_m \odot K_1$), $m \geq 2$ is Proper d-lucky, define the function $l: R(G) \rightarrow N$ to label the vertices as follows:

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Case(i):$m = 1(mod2)$

(i) $l(r_1) = l(r_m) = 1$,

(ii) $2 \leq i \leq m-1$, m≥ 3, $l(r_i) = \{2,\ i \equiv 0(mod2)\ 1,\quad otherwise$,

(iii) $1 \leq i \leq m$, $l(r_{i+m}) = \{1,\ \ 0(mod2)\ 2,\ otherwise$,
$l(r_{2m+(2i-1)}) = \{1,\ i \equiv 0(mod2)\ 2,\quad otherwise$, $l(r_{2m+2i}) = \{1, i \equiv 0(mod2)\ 2,\quad otherwise$.

Case(ii): $m = 0(mod2)$

(i) $l(r_1) = 1$, $l(r_m) = 2$,

(ii) $2 \leq i \leq m-1, m \geq 3, l(r_i) = \{2, i \equiv 0(mod2)\ 1,\quad otherwise$,

(iii) $1 \leq i \leq m, l(r_{i+m}) = \{1, i \equiv 0(mod2)\ 2,\quad otherwise$,
$l(r_{2m+(2i-1)}) = \{1, i \equiv 0(mod2)\ 2,\quad otherwise$, $l(r_{2m+2i}) = \{1, i \equiv 0(mod2)\ 2,\quad otherwise$.

(i) $d(r_1) = d(r_m) = 4$,

(ii) For $2 \leq i \leq m-1$, $d(r_i) = 5$,

(iii) For $m+1 \leq i \leq 4m, d(r_i) = 1$.

Case(i): Therefore,

(i) $c(r_1) = c(r_n) = 12$,

(ii) $2 \leq i \leq m-1, m \geq 3, c(r_i) = \{10, i \equiv 0(mod2)\ 15,\quad otherwise$,

(iii) $1 \leq i \leq m, c(r_{i+m}) = c(r_{2m+(2i-1)}) = c(r_{2m+2i}) = \{3, i \equiv 0(mod2)\ 2,\quad otherwise$,

Case(ii): Therefore,

(i) $c(r_1) = 12, c(r_n) = 8$,

(ii) $2 \leq i \leq m-1, m \geq 3, c(r_i) = \{10, i \equiv 0(mod2)\ 15,\quad otherwise$,

(iii) $1 \leq i \leq m, c(r_{i+m}) = c(r_{2m+(2i-1)}) = c(r_{2m+2i}) = \{3, i \equiv 0(mod2)\ 2,\quad otherwise$.

Clearly, $c(r_i) \neq c(r_{i+m})$, $c(r_i) \neq c(r_{2m+(2i-1)}), c(r_i) \neq c(r_{2m+2i})$, for any two adjacent vertices of Twig($P_m \odot K_1$).Therefore Twig($P_m \odot K_1$) admits Proper d-lucky labeling with $\eta_{dl}(Twig(P_m \odot K_1)) = 2$.

EXAMPLE 3.2:

Proper d-Lucky labeling for Twig($P_m \odot K_1$) for m = 4 and m = 5 is shown in the figure 3.2.1 and 3.2.2 respectively.

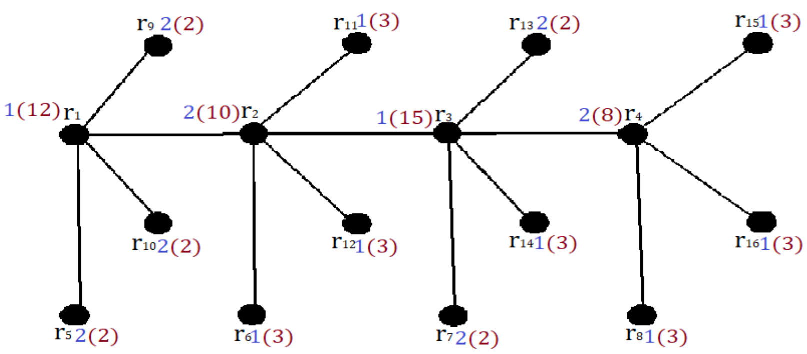

Figure 3.2.1

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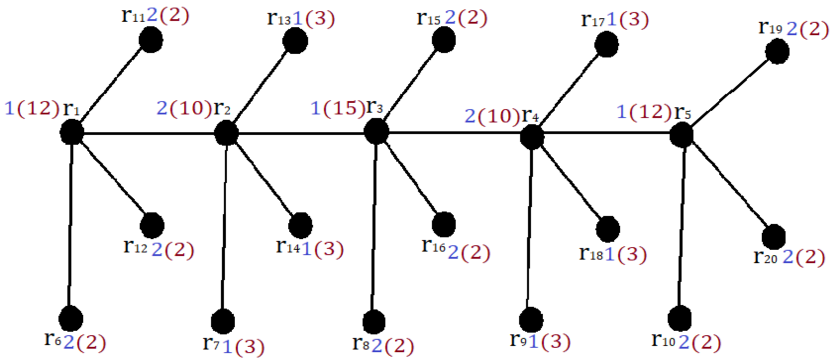

Figure 3.2.2

THEOREM 3.3:

Twig$(P_m \odot K_1)$, $m \geq 2$ admits Tribonacci Product Cordial labeling.

Proof:

From the structure of Twig$(P_m \odot K_1)$, $m \geq 2$. It is clear that Twig$(P_m \odot K_1)$ has $4m$ vertices and $4m-1$ edges.
Define the function $h: R(G) \rightarrow \{0,1\}$ to obtain the vertex lables as follows:
For $1 \leq i \leq m$

(i) $r_i = \{ T_{4I-2}\}$
(ii) $r_{2m+(2i-1)} = \{T_{4I-3}\}$
(iii) $r_{i+m} = \{ T_{4I}\}$
(iv) $r_{2m+2i} = \{T_{4I-1}\}$

Clearly, $R_h(0) = 2m$, $R_h(1) = 2m$,
Therefore, $|R_h(0) - R_h(1)| = |2m - 2m| = 0 \leq 1$
Define the function $h^*: B(G) \rightarrow \{0,1\}$ to obtain the edge lables as follows:
For $1 \leq i \leq m-1$

(i) $r_i r_{i+1} = 1$

For $1 \leq i \leq m$

(i) $r_i r_{2m+(2i-1)} = 1$
(ii) $r_i r_{i+m} = r_i r_{2m+2i} = 0$

Clearly, $B_{h^*}(0) = 2m-1$, $B_{h^*}(1) = 2m$,
Therefore, $|B_{h^*}(0) - B_{h^*}(1)| = |(2m-1) - 2m| = 1 \leq 1$
Hence the conditions $|R_h(0) - R_h(1)| \leq 1$ and $|B_{h^*}(0) - B_{h^*}(1)| \leq 1$ are satisfied.
Therefore, Twig$(P_m \odot K_1)$, $m \geq 2$ admits Tribonacci Product cordial labeling.

EXAMPLE 3.3:

Tribonacci Product Cordial labeling for Twig$(P_m \odot K_1)$ m = 4 is shown in the figure 3.3.1 respectively.

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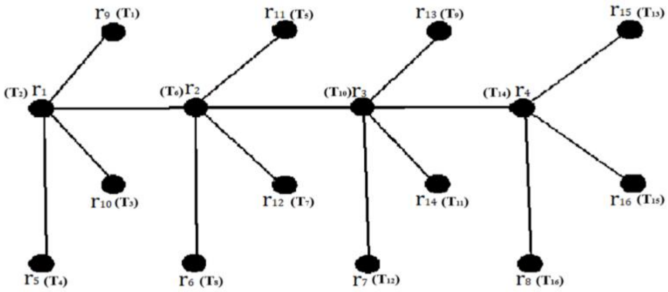

Figure 3.3.1

THEOREM 3.4:

Twig($P_m \odot K_1$), $m \geq 2$ admits Gaussian Antimagic labeling.

Proof:

From the structure of Twig($P_m \odot K_1$). It is clear that Twig($P_m \odot K_1$) has $4m$ vertices and $4m - 1$ edges. Define the function $h: R(G) \to \{a + ib \ / \ a, b \in \ N\ \}$ to obtain the vertex lables as follows:

For $1 \leq j \leq 4m$

$r_j = 1 + ri$

Define the function $h^* : B(G) \to N \text{ such that } h^*(ru) = |h(r)|^2 + |h(u)|^2$ to obtain the edge lables as follows:

(i) $h^*(r_j r_{j+1}) = 2j^2 + 2j + 3 \quad : 1 \leq j \leq m - 1$

(ii) $1 \leq j \leq m$

$$h^*(r_j r_{j+m}) = 2j^2 + m^2 + 2mj + 2$$

$$h^*(r_j r_{2m+(2j-1)}) = 5j^2 + 4m^2 + 8mj + 3 - 4j - 4m$$

$$h^*(r_j r_{2m+(2j)}) = 5j^2 + 4m^2 + 8mj + 2$$

Hence in which all the elements are distinct.

Therefore, Twig($P_m \odot K_1$), $m \geq 2$ admits Gaussian Antimagic labeling.

EXAMPLE 3.4:

Gaussian Antimagic labeling for Twig($P_m \odot K_1$), $m = 4$ is shown in the figure 3.4.1 respectively.

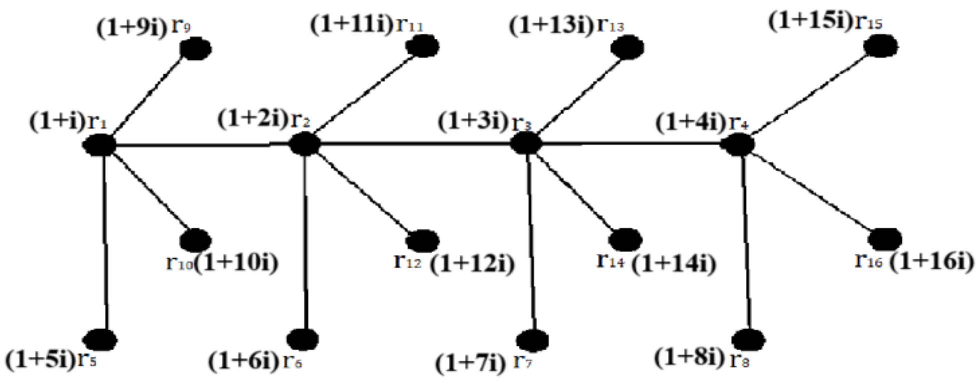

Figure 3.4.1

CONCLUSION:

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In this paper, we have proved the existence of Proper Lucky Labeling, Proper d-lucky Labeling, Tribonacci Product Cordial Labeling and Gaussain Anti-magic labeling for Twig$(P_m \odot K_1)$, $m \geq 2$.

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