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CHANGE IN THE TEETH BENDING STRESS AND HERTZIAN CONTACT STRESS ON THE TEETH IN A PLANETARY GEAR REDUCER DEPENDING ON THE NUMBER OF REVOLUTION OF THE INPUT SHAFT

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ABSTRACT
The planetary reducer is used to reduce the number of revolutions of the drive shaft in relation to the number of revolutions of the driven shaft. In the considered reducer, the power is supplied through the central gear (Sun) and it is the drive, the shaft - carrier on which the satellite gears are mounted is the output shaft. The external gear is stationary. In a planetary reducer with constant input power, changing the number of revolutions of the input shaft cause a change in the stresses in the roots of the teeth and the Hertzian contact stress on the gear pair formed from the central and the satellite gear. In this paper, it is analyzed changing of these stresses depending on number of revolutions of the input shaft. The change in the ratio between the Hertzian contact stresses and teeth bending stress also depending on the number of revolutions of the input shaft.

Keywords:
Planetary reducer, Bending stress, Contact stress, Transmission rate

INTRODUCTION
Planetary gear trains are widely used in the different fields of engineering, particularly in the mechanical engineering. They are used for the smallest and the largest ratios, have high efficiency, low weight, small dimensions. Because the diminished dimension they have small mass moment of inertia, lower internal dynamic loads and a quieter operation of the gear train, reduced material consumption and a light construction. Planetary gear trains (PGTs) are most often coaxial. They are designed in such a way that they possess at least one or more gear wheels, and these are called planets mounted on the so-called carrier, which perform a double rotation around their geometrical axis and together with the same axis around the main (the central) geometrical gear train axis. Thus, the complicated motion of the planets comprises a vector summation of both rotations-the revolution performed by the carrier H and the relative one (spin), performed by each planet 2 with respect to the carrier.

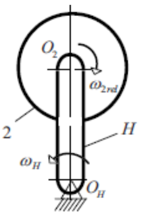

Fig.1. Revolution and relative rotation (spin) of the planets

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This typical planet motion explains the name of these gear trains, whose motion resembles the motion of the planets around the central gear (Sun), and this is their typical distinctive kinematics characteristic. Planets are meshing with one or two central gear wheels with external or internal teeth.

Planetary transmissions have a number of capabilities in solving certain tasks in the field of power transmission: Achieving a constant transmission ratio they can work as a reducer -(with one degree of freedom), it is most commonly used when the ring is fixed and rarely as a multiplier - (with one degree of freedom)[1].

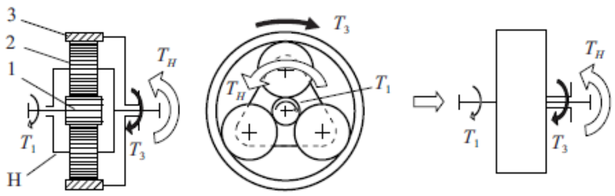

Fig. 2. Simple planetary gear train. Central gear, Sun (1), Satelite gears (2), Ring gear (3), Carrier (H)

Each simple PGT, whether positive-ratio or negative-ratio one, possesses three shafts, coming out of the gear that means it is a three-shaft gear train. The three external shaft torques (T1, T3 and T_H) of all the simple PGTs are in equilibrium [1],[2]

$$\sum T_i = T_1 + T_3 + T_H = 0 \qquad (1)$$

In this manuscript kinematic analysis is performed with the superposition method-Swamp rule. According to this method, individual elements of the planetary transmission are observed in different specified movements in few set movements. Number of teeth on the gear one is z_1, number of teeth on the career is z_2 and number of teeth on the gear 3 is z_3. Number of revolutions of the planetary reducer gears are [3], [4]

$$n_2 = -\frac{z_1}{z_2} * n_1 + \left(\frac{z_1}{z_2} + 1\right) * n_s; \qquad (2)$$

$$n_3 = -\frac{z_1}{z_2} * n_1 + \left(\frac{z_1}{z_3} + 1\right) * n_s; \qquad (3)$$

The number of rotations of gear 1: $n_1 = n_{input}$
The number of rotations of carrier H: $n_s = n_{output}$
The number of rotations of gear 3 is n_3
The direction of rotation of the output shaft, is opposite to the direction of rotation of the input shaft, where the central gear with external teeth (sun gear) is located.

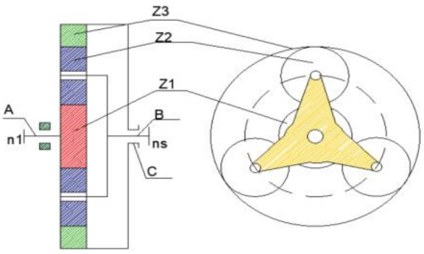

Fig. 3: Planetary gear train 1AI

Since the central gear with internal teeth (ring gear) in this manuscript will be the reaction member, its number of rotations will be equal to zero. The overall transmission ratio will be: $i = n_1 / n_2$.

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Using coaxiality condition, which guarantees equal spacing between the first set (sun gear and planetary gears) and the second set (planetary gears with ring gear) the number of teeth on the central gear with internal teeth is [1]:

$$z_3 = z_1 + 2z_2 \tag{4}$$

The transmission ratio between gears 1 and 2 is:

$$i_{12} = -\left(\frac{z_2}{z_1}\right) \tag{5}$$

The transmission ratio between gears 2 and 3 is:

$$i_{23} = \frac{z_3}{z_2} \tag{6}$$

The actual standard transmission ratio is:

$$i_0' = i_{12} * i_{23} \tag{7}$$

The total power transmitted by the planetary transmission consists of two components: the power at the coupling P_k and the force due to the rolling of the teeth P_z, also known as kinematic power.

Input shaft is shaft A

$$P_A = \omega_A * T_A, \text{ [W]} \tag{8}$$

Angular velocity of the shafts:

$$\omega_A = n_A * \frac{\pi}{30} \text{ [s}^{-1}\text{]} \tag{9}$$

$$\omega_B = n_B * \frac{\pi}{30} \text{ [s}^{-1}\text{]} \tag{10}$$

$$\omega_C = n_C * \frac{\pi}{30} \text{ [s}^{-1}\text{]} \tag{11}$$

Torque of the input shaft A:

$$T_A = \frac{P_A}{\omega_A} \tag{12}$$

Torque of shaft B:

$$T_B = -(T_A + T_C) = -(1 - i_0 * \eta_0) * T_A \tag{13}$$

Torque of shaft C:

$$T_C = -i_0 * \eta_0 * T_A \text{ [Nm]} \tag{14}$$

Due to the division of the power on P_k and P_Z, the power on the shaft A is:

$$P_{ZA} = \omega_{AS} * T_A \tag{15}$$

$$P_{ZA} = (\omega_{A0} - \omega_{s0}) * T_A \tag{16}$$

$$P_{KA} = \omega_{S0} * T_A \tag{17}$$

The powers on shaft B are:

$$P_{ZB} = \omega_{BS} * T_B = (\omega_{B0} - \omega_{s0}) * T_B \tag{18}$$

$$P_{KB} = \omega_{S0} * T_B \tag{19}$$

The powers on shaft C are:

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$$P_{ZC} = \omega_{CS} * T_C = (\omega_{C0} - \omega_{s0}) * T_C \qquad (20)$$

$$P_{KC} = \omega_{S0} * T_C \qquad (21)$$

Module of the gear pairs z_1-z_2 is calculated using the formula [1], [5],[6]:

$$m \geq \sqrt[3]{\frac{2 * T_1}{\lambda * z_1 * \sigma_{FP}} * Y_{Fa} * Y_{Sa} * K_{F\alpha} * K_{F\beta} * K_v * K_a} \qquad (22)$$

Y_{Fa} - tooth shape factor, Y_{Sa} – stress correction factor, $K_{F\alpha}$ - load distribution factor in the frontal cross-section for stress at the tooth root, $K_{F\beta}$ - load distribution factor along the length of the tooth for stress at the tooth root, K_v - distribution factor of the peripheral force along the engagement of central gears with planetary gears, K_a - factor for uniform loading of the machine in continuous operation with the electric motor, λ - tooth width factor and σ_{FP} - root bending stress. As a material for the manufacture of the central gear with external teeth, it is choiced induction-hardened steel.
Because there are three planetary gears in the planetary transmission, the total power is divided into 3 parts:

$$P_1 = \frac{P_{ZA}}{N} \qquad (23)$$

The torque at the point of contact between gear 1 and one of the planetary gears 2 will be:

$$T_1 = \frac{P_1}{\omega_1} \qquad (24)$$

Calculation of the orientation module of the gear pair $z_1 - z_2$

Because this planetary transmission is of the 1AI type, all gears need to have the same module to achieve mutual engagement. It was necessary to calculate the orientation module of the gear pair, and then a module that meets the requirements for both gear pairs was selected [6].

$$m \geq \sqrt[3]{\frac{2 * T_2}{\lambda * z_2 * \sigma_{FP}} * Y_F * Y_\varepsilon * K_{H\alpha} * K_{H\beta} * K_v * K_I} \qquad (25)$$

$K_{H\alpha}$ – transverse load factor for contact stress, $K_{H\beta}$ – face load factor contact stress

Because there are 3 planetary gears in the planetary transmission, the total power is divided into 3 parts:

$$P_2 = \frac{P_{ZC}}{N} \qquad (26)$$

The torque at the point of contact between gear 3 and one of the planetary gears 2 will be:

$$T_2 = \frac{P_2}{\omega_2} \qquad (27)$$

Where the angular velocity of the planetary gear is ω_2, and it is:

$$\omega_2 = \frac{\pi * n_2}{30} \qquad (28)$$

Dimensions of the gears

The width of the teeth is determined from the expression for the tooth width factor [7], [8].
The width of the teeth is $\lambda = b/m$

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The center distance between the two gear pairs is equal and is:

$a_{12} = a_{23}$, $\quad a_{12} = \frac{(z_1+z_2)*m/2}{2}$ (29)

Diameters of gear 1 (central gear with external teeth)
Reference diameter: $d_1 = m * z_1$, Tip diameter: $d_{a1} = d_1 + 2m$, Root diameter: $d_{f1} = d_1 - 2{,}5m$
Diameters of gear 2 (planetary gears)
Reference diameter: $d_2 = m * z_2$,
Tip diameter: $d_{a2} = d_2 + 2m$, Root diameter: $d_{f2} = d_2 - 2{,}5m$
Diameters of gear 3 (central gear with internal teeth)
Reference diameter: $d_3 = m * z_3$, Tip diameter: $d_{a3} = d_3 + 2m$, Root diameter: $d_{f3} = d_3 - 2{,}5m$

Inspection of the teeth root bending stress

Gear pair $\boldsymbol{z_1 - z_2}$
The teeth root bending stress is calculated according to the equation :

$$\sigma_{F1} = \frac{F_{tw1}}{b * m} * Y_{Fa} * Y_{Sa} * Y_B * K_{F\alpha} * K_{F\beta} * K_v * K_a \quad (30)$$

The force F_{tw1} is:

$$F_{tw1} = \frac{2*T_1}{dw_1} \quad (31)$$

Gear pair $\boldsymbol{z_2 - z_3}$

$$\sigma_{F2} = \frac{F_{tw2}}{b * m} * Y_{Fa} * Y_{Sa} * Y_B * K_{F\alpha} * K_{F\beta} * K_v * K_a \quad (32)$$

The force F_{tw2} is:

$$F_{tw2} = \frac{2 * T_2}{dw_2} \quad (33)$$

Inspection of the Hertzian contact stress a

Gear pair $z_1 - z_2$
The contact stress along teeth profile is calculated using Hertz formula :

$$\sigma_{H1} = Z_H * Z_\varepsilon * \sqrt{\frac{u+1}{u} * \frac{F_{tw1}}{b * d_{w1}} * K} \quad (34)$$

$u = z_2 / z_1,$

Gear pair z_2 - z_3
The contact stress along teeth profile is calculated using Hertz formula:

$$\sigma_{H2} = Z_H * Z_E * \sqrt{\frac{u-1}{u} * \frac{F_{tw2}}{b * d_{w2}} * K} \quad (35)$$

$u = z_3 / z_2$

$$Z_H = \sqrt{\frac{2}{\cos(\alpha w) * tg\alpha w}} \quad (36)$$

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Z_H – zone factor . For a standard spur gear pressure angle is 20° and Z_H=2,5, Z_E – elastic coefficient. For the common match of steel pinion and steel gear, the elastic coefficient is 189.8 N/mm^2.

CALCULATION AND RESULTS

First the planetary reducer transmission ratios were calculated for the given values z_1=21,25,29,33 and 37, z_2=85 was all time, z_3=191,195,199,203 and 207 respectively, and power P=7.200 [W]. Then, Hertzian contact strees of the gear pair z_1-z_2 were calculated using (34) for different numbers of revolutions of the input shaft 1, from 600 to 4.200 (600, 900, 1200, 1500, 1800, 2100, 2400, 2700, 3000, 3300, 3600, 3900 and 4200) number of revolutions per minute. Obtained values are shown in table 1.

Z_1	Number of revolutions per minute of the Input Shaft of Planetary Reducer											
	600	**900**	**1200**	**1500**	**1800**	**2100**	**2400**	**2700**	**3000**	**3200**	**3500**	**4200**
21	523,42	427,37	370,12	331,04	302,2	279,78	261,71	246,74	234,08	223,19	213,69	205,3
25	444,26	362,73	314,14	280,97	256,49	237,46	222,13	209,42	198,68	189,43	181,37	174,25
29	386,89	315,89	273,57	244,69	223,37	206,8	193,44	182,38	173,02	164,97	157,95	151,75
33	343,39	280,38	242,81	217,18	198,26	183,55	171,7	161,88	153,57	146,42	140,19	134,69
37	309,27	252,52	218,69	195,6	178,56	165,31	154,64	145,79	138,31	131,87	126,26	121,31

Table 1: Hertzian contact stress of the gear pair z_1-z_2.

Based on the values from table 1 for z_1=29 first is drawn the diagram presented in the Figure 4. This curve actually determines the gear pair z_1-z_2, Hertzian contact stress depending on the number of revolutions of the input shaft n_1. The equation of this curve is $y=9476{,}6x^{-0.5}$ and it was calculated using approximation method. Later, on the values from table 1 are drawn the diagrams presented in the Figure 5. Thece curves actually determines the gear pair z_1-z_2, Hertzian contact stress depending on the number of revolutions of the input shaft n1 when z_1=21,25,29,33,37.

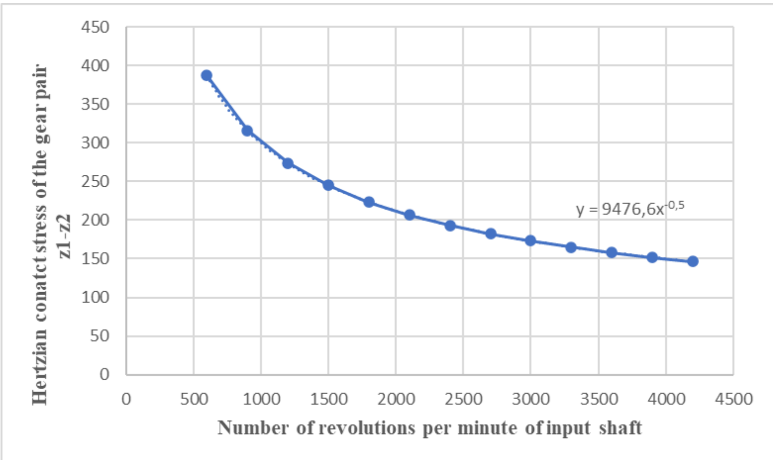

Fig. 4. Hertzain contact stress for z_1=29 and different number of revolutions of the input shaft

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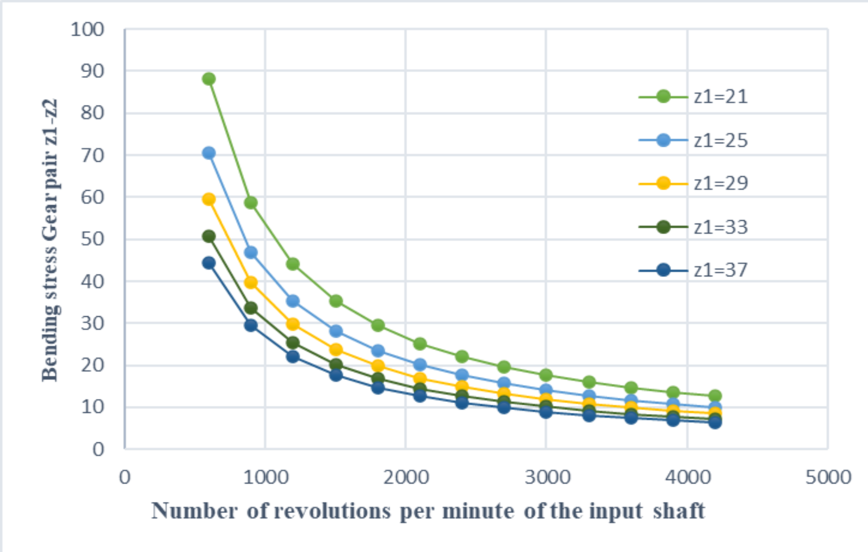

Fig. 5. Hertzain contact stress for different number of revolutions of the input shaft

Z_1	Number of revolutions per minute (rpm) of the Input Shaft of Planetary Reducer											
	600	**900**	**1200**	**1500**	**1800**	**2100**	**2400**	**2700**	**3000**	**3200**	**3500**	**4200**
21	88,16	58,77	44,08	35,26	29,39	25,19	22,04	19,59	17,63	16,03	14,69	13,56
25	70,49	46,99	35,25	28,2	23,5	20,14	17,63	15,66	14,1	12,82	11,75	10,84
29	59,47	39,65	29,74	23,79	19,82	16,99	14,87	13,22	11,89	10,81	9,91	9,15
33	50,6	33,73	25,3	20,24	16,87	14,46	12,65	11,24	10,12	9,2	8,43	7,78
37	44,32	29,55	22,16	17,73	14,77	12,66	11,08	9,85	8,86	8,06	7,39	6,82

Table 2 Bending stress in the rooth of the teeth of the gear pair z_1-z_2.

Based on the values from table 2 for z_1 =29 is drawn the diagram presented in the Figure 6. This curve actually determines the gear pair z1-z2, bending of the teeth of gear pair $z_1 - z_2$. depending on the number of revolutions of the input shaft n1. The equation of this curve is $y=35682x^{-1}$ and it was calculated using approximation.
Later on the values from table 2 are drawn the diagrams presented in the Figure 7. These curves actually determines the gear pair z_1-z_2, bending stress of in the rooth of the teeth depending on the number of revolutions of the input shaft n1 when z_1=21,25,29,33,3.

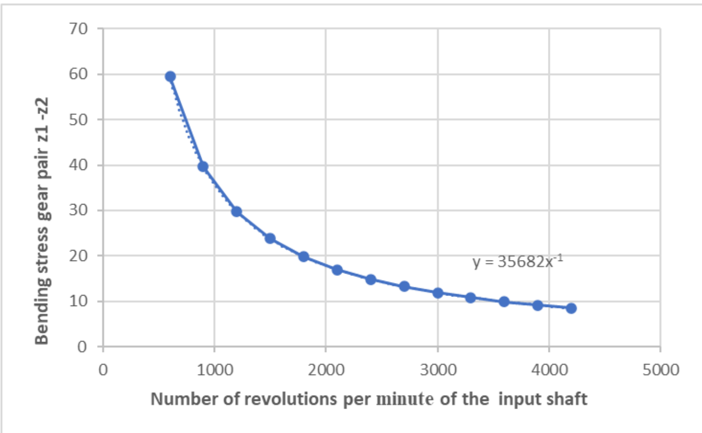

Fig. 6. Bending stress teeth for different number of revolutions of the planetary input shaft

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Based on the values from table 2 are drawn and the diagrams presented in the Figure 7. This curves actually determines the gear pair z_1-z_2, bending stress on the rooth of the teeth depending on the number of revolutions of the input shaft n_1 when z1=21,25,29,33,37.

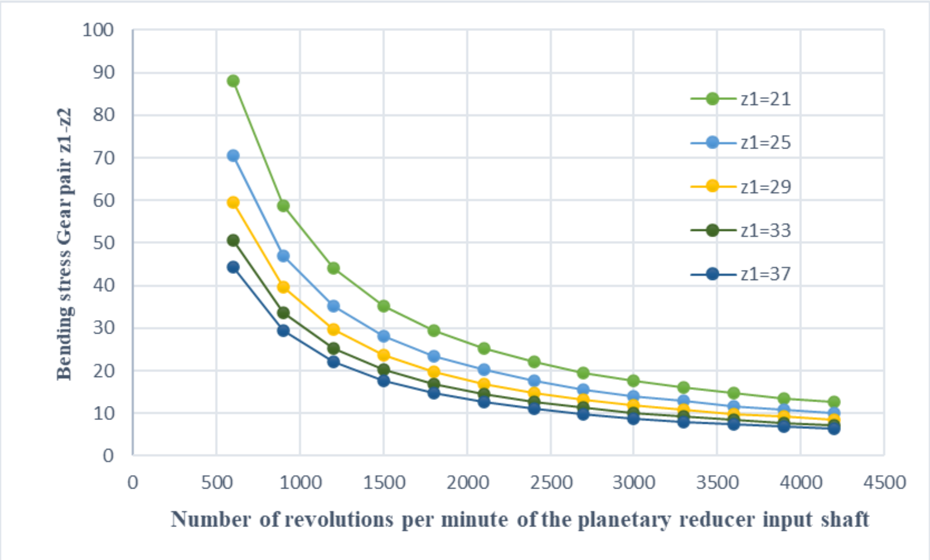

Fig.7. Bending stress of the gear pair (z_1-z_2) tooth for different number of revolutions of the planetary input shaft for different values of z_1

RESULTS AND DISCUSION

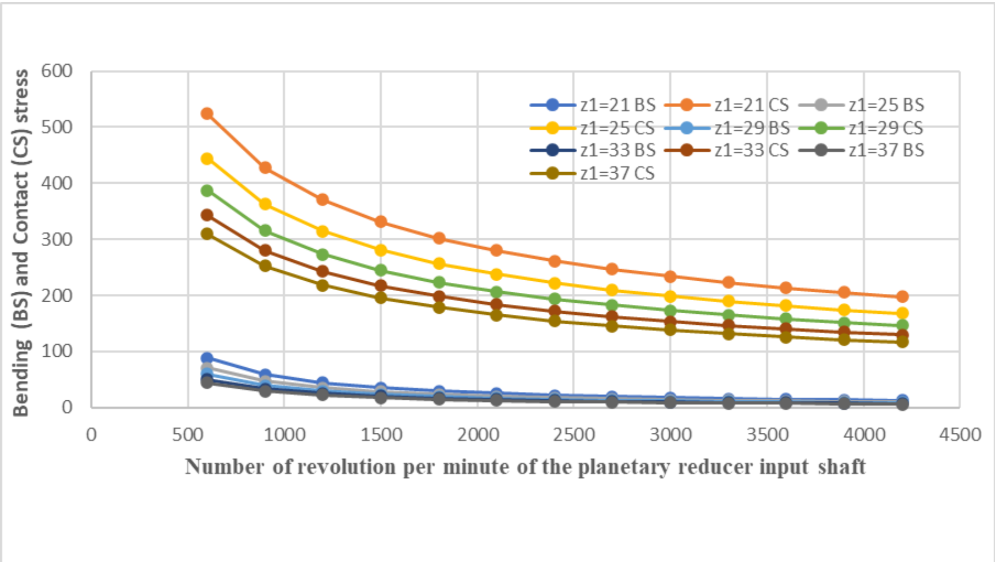

Fig. 8. Bending and Hertzian contact stresses of the gear pair (z_1-z_2) tooth for different number of revolutions of the planetary input shaft for different values of z_1

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Comparing the data from Table 1 and Table 2, for z_1=29, it can be seen that the Hertzian contact stressct in this case is numerically much greater than the bending stress in the root of the teeth. For example, for n_1 = 600 [rpm], the contact stress is 444.26 [N/mm^2], and the bending stress in the root of the teeth is 70.49 [N/mm^2]. With an increasing in the number of rotation per minute, both stresses have a decreasing trend, but in percentage terms, the bending stress in the root of the teeth in the interval from 600 [rpm] to 1800 [rpm] decreases much faster. From 70.49 [N/mm^2] for 600 [rpm] to 23.5 [N/mm^2] for 1800 [rpm], i.e. it decreases by about 67%, while the Hertzian contact stress contact increase from 444.26 [N/mm^2] for 600 [rpm] to 256.49 [N/mm^2] or decreases by about 42%. With further increasing of the number of the rotations per minute, both stresses continue to decrease, but throughout the entire time, the Hertzian contact stress is numerically several times greater than the bending stress in the root of the teeth while Hertzian contact stress decreases in percentage much slower than the bending stress in the root of the teeth.
From Fig.8 and Table 1 and Table 2 it is clear that the analyses is the same and for z_1 = 21, 25,29,33 and 37.

It is calculated and then represented on Figure 9 ratio between Hertzian contact stress and bending stress in the root of the teeth. From Figure 9 we can see that for a constant number of teeth, as the revolutions of input shaft speed increases, the Hertzian contact stress also increase and that when the number of revolutions of the input drive shaft is constant this ratio increases with increasing number of the teeth. However, but this increase is percentage is much smaller than in the previous case when number of teeth was constant and nymber of revolution was changing.

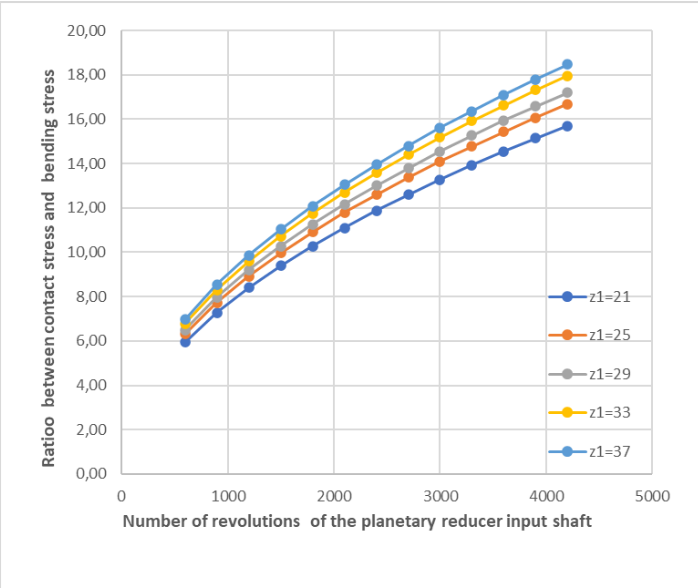

Fig. 9. Ratio between Hertzian contact stress and Bending stress in the root of the teeth

CONCLUSION

From the presented results it can be concluded that when using planetary reducers, operating at constant power on the input shaft in the reducer, for a given value of one of the analyzed stresses, the bending stress in the rooth of the teeth or the Hertzian contact stress, the number of teeth of the gear pair z_1 - z_2 can be selected depending on the operating speed of the input shaft and vice versa, for a given number of teeth, the number of revolutions of the input shaft can be selected.

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Also, from the last diagram (Fig.9) for a certain number of revolutions, the ratio between the Hertzian contact stress and bending stress in the rooth on the teeth can be determined.
When machines start working, the number of revolutions of the input shafts constantly changes from zero to the intended operating number of revolutions of the shaft. During this period, the number of revolutions is constantly increasing, and in this time interval the highest stresses and most of the problems occur..

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