Vol-07 Issue 12, December- 2023
ISSN: 2456-9348
Impact Factor: 6.736

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

# IMPACT OF THE INPUT SHAFT NUMBER OF REVOLUTIONS ON THE GEAR PAIR TEETH BENDING STRESS IN A PLANETARY REDUCER

**Sasko Milev[*1],**
**Darko Tasevski[2]**
**Mevludin Shabani[3]**
**Lazar Jovanov[4]**
**Zoran Dimitrovski[5]**
[*1,5] Faculty of Mechanical Engineering, Goce Delcev University, Stip, North Macedonia
Krste Misirkov,10A, 2000, Stip,
[2]Faculty of Mechanical Engineering, Ss Cyril and Methodius University in Skopje, North Macedonia
[3]Faculty of Mechatronics, University of Business and Technology, Prishtina, Kosovo,
[4] Ruen-Inox Automobile, Kocani, North Macedonia

## ABSTRACT
This paper aimed to examine the possibilities to improve the characteristics of planetary reducers, considering their wide application in mechanical engineering. For this purpose, it was first investigated how the change in the speed of the input shaft of the planetary reducer affects the bending stress in the roots of the teeth of the gear pairs that are formed by the teeth of the driving shaft and the satellite gears, at a constant number of teeth and a constant transmission ratio of the planetary gear. Then it was investigated whether and how the change in the number of teeth on the driving shaft affects the bending stress of the teeth.

**Keywords:**
Planetary Reducer, Bending Stress, Gear Pair Teeth, Transmission Rate,

## INTRODUCTION
Mechanical transmission is transfer of power or motion from the driving to the driven element through a mechanical assembly called a transmission. Transmissions consist of driving and driven rotate elements, transfer is achieved through their direct or indirect contact, for example, using a gear pair, belt, chain. Changing the number of rotations is accomplished by altering the diameter of the driving and the driven elements. Main characteristic of mechanical transmissions is their transmission ratio. When transmission ratio is greater than 1 ($i > 1$), the number of rotations decreases, the torque increases, and the transmission is called a reducer. Conversely, when the transmission ratio is less than 1 ($i < 1$), the number of rotations increases, the transmission is called a multiplier. Planetary transmissions belong to the group of mechanical transmissions, i.e., gear mechanical transmissions and the mechanical transfer is achieved by at least one element (the satellite gear), rotating not only around its own axis but also around another axis. These mechanical transmissions find application when is a need for transfer of large forces and a high number of rotations, with the imposed requirement for a small volume and weight of the transmission. In planetary transmissions, the force is distributed among multiple planetary gears [1] [2].
Advantages of planetary transmissions in relation to non-planetary ones are; high compactness expressed through small overall dimensions and low volume, low peripheral speeds, and thus a small level of noise, small mass, two or three times less than that of non-planetary transmissions, using the principle of separation of torque (between several planetary gears), very high efficiency–higheast of all transmissions, relatively good cinematic abilities, relatively low noise level due to: lower peripheral speeds, smaller dynamic forces due to smaller dimensions, high transmission ratio within

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

one degree, etc. Although the large number of advantages, there are also disadvantages: geometric calculations are complex, the uneven distribution of load between planets is a complex and specific problem whose solution is not easy, the teeth of the planetary gears are rolled less favorably than in other gears, they have a complex construction, and a high risk of a defect of the entire transmission, high production technology is needed(due to the imposed quality requirements). Planetary transmissions have a number of capabilities in solving certain tasks in the field of power transmission: 1.Achieving a constant transmission ratio $i = const$ can work as a reducer -$|i| > 1$ (with one degree of freedom), it is most commonly used when the ring is fixed, i.e. $\omega_3 = 0$, less often when $\omega_1 = 0$, and rarely when $\omega_s = 0$, and as a multiplier - $|i| < 1$(with one degree of freedom) it is commonly used when $\omega_3 = 0$ in hydro turbines, less often when $\omega_1 = 0$, and rarely when $\omega_s = 0$. 2.Implementation of stepwise change of transmission ratio in single and double drives. 3. Implementation of continuous change in transmission ratio, differential (with two degrees of freedom), achieving a high angular velocity and other application. The designation of planetary gear systems is done using the numbers 1 and 2, and the letters A and I. The numbers 1 and 2 indicate whether the gear system is single-stage or two-stage, while the letter A denotes central gears with external teeth (sun gear), and the letter I denotes central gears with internal teeth (ring gear). Simple planetary gear systems are of type 1AI, and they have a single-stage planetary set with one central gear with external teeth (sun gear) and one central gear with internal teeth. Planetary gear systems can be used as reducers, multipliers (with one degree of freedom), and differentials (with two degrees of freedom), depending on their construction [3].

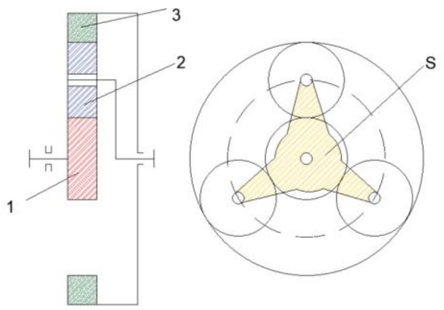


***Fig. 1: Planetary gear train 1AI***

This planetary gear system consists of the following main components: central gear with external teeth 1(sun gear), planetary gears 2, central gear with internal teeth 3(ring gear), and carrier S. The central gear with external teeth is the driving element and is attached to the input shaft. It is coupled with the planetary gears, which perform two rotations, one around their axis and the other around the axis of the central gears. The central gear with internal teeth is in contact with the planetary gears but does not undergo rotational movement as it serves as a reaction member in this case. The carrier S is attached to the shafts of the planetary gears, and the output shaft is inserted into it, transmitting rotational movement to the output shaft as a result of the rotation of the planets around the axes of the central gears. Since this is a reducer, the number of rotations of the output shaft will be less than the number of rotations of the input shaft.

## OBJECTIVES

The main objective of the study is to determine the impact of the change of the number of revolutions of the planetary reducer input shaft on the gear pair($z_1$-$z_2$) teeth bending stress , when the number of driving shaft teeth is unchanged. Then the impact of the change of the number of driving gear teeth on the bending stress of the of the gear pair teeth is analyzed.

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

## METHODOLOGY

### Kinematic analysis of 1AI planetary gear train

The kinematic analysis is performed with the superposition method-Swamp rule. According to this method, individual elements of the planetary transmission are observed in different specified movements in few set movements. Number of rotations and the number of revolutions of the planetary reducer gears are given in Table 1 [4].

***Table 1: Kinematic Analysis of Planetary Gear Train***

| Number of movements | Gear 1 $(z_1)$ | Gear 2 $(z_2)$ | Gear 3 $(z_3)$ | Carrier S |
|---|---|---|---|---|
| 1 | +1 | +1 | +1 | +1 |
| 2 | -1 | $\frac{z_1}{z_2}$ | $\frac{z_1}{z_2} * \frac{z_2}{z_3} = \frac{z_1}{z_3}$ | 0 |
| Sum of the first two motions | 0 | $1 + \frac{z_1}{z_2}$ | $1 + \frac{z_1}{z_3}$ | +1 |
| 3 | $+n_1$ | $-\frac{z_1}{z_2} * n_1$ | $-\frac{z_1}{z_3} * n_1$ | 0 |
| 4 | 0 | $\left(\frac{z_1}{z_2} + 1\right) * n_s$ | $\left(\frac{z_1}{z_3} + 1\right) * n_s$ | $+n_s$ |
| Sum of the 3rd and 4th motions | $+n_1$ | $-\frac{z_1}{z_2} * n_1 + \left(\frac{z_1}{z_2} + 1\right) * n_s$ | $-\frac{z_1}{z_3} * n_1 + \left(\frac{z_1}{z_3} + 1\right) * n_s$ | $+n_s$ |

The number of rotations of gear 1: $n_1 = n_{input}$
The number of rotations of carrier s: $n_s = n_{output}$
From the kinematic analysis (Table 1), expressions are derived to determine the number of rotations of gear 2 (planetary gear) and the number of rotations of gear 3 (ring gear)

$$n_2 = -\frac{z_1}{z_2} * n_1 + \left(\frac{z_1}{z_2} + 1\right) * n_s; \qquad n_3 = -\frac{z_1}{z_2} * n_1 + \left(\frac{z_1}{z_2} + 1\right) * n_s;$$

The overall transmission ratio can be obtained from the ratio of the number of rotations at the input and output of the planetary gear train. The direction of rotation of the output shaft, is opposite to the direction of rotation of the input shaft, where the central gear with external teeth (sun gear) is located.

IJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

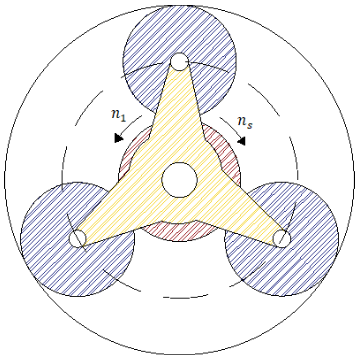


***Fig. 2 : Direction of rotation of the carrier S and gear 1***

Since the central gear with internal teeth (ring gear) will be the reaction member, its number of rotations will be equal to zero. The overall transmission ratio will be: $i= n_1/ n_2$

**Coaxiality condition**

Using coaxiality condition, which guarantees equal spacing between the first set (sun gear and planetary gears) and the second set (planetary gears with ring gear) the number of teeth on the central gear with internal teeth is:

$$z_3 = z_1 + 2z_2$$

In this expression, it is assumed that the gears will have straight teeth and no profile shifting (x=0). The number of teeth on the central gear with external teeth (sun gear) and planetary gears are chosen arbitrarily. Due to the high number of rotations at the input, it is preferable to choose a smaller number of teeth of sun gear to reduce the size and, consequently, the mass of the gear. Once we have the numbers of all teeth, we can establish the transmission ratios between individual pairs of gears. In the case where the direction of rotation of the driven gear is opposite to the direction of rotation of the driving gear, a negative sign is assigned to the transmission ratio [5].

The transmission ratio between gear 1 (sun gear) and gear 2 (satellite gears) is:

$$i_{12} = -\left(\frac{z_2}{z_1}\right)$$

The transmission ratio between gear 2 (planetary gears) and gear 3 (ring gear) is:

$$i_{23} = \frac{z_3}{z_2}$$

Hence, the actual standard transmission ratio is:

$$i_0' = i_{12} * i_{23}$$

**Sizing of the gears**

The total power transmitted by the planetary transmission consists of two components: the power at the coupling $P_k$ and the force due to the rolling of the teeth $P_z$, also known as kinematic power [4].

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

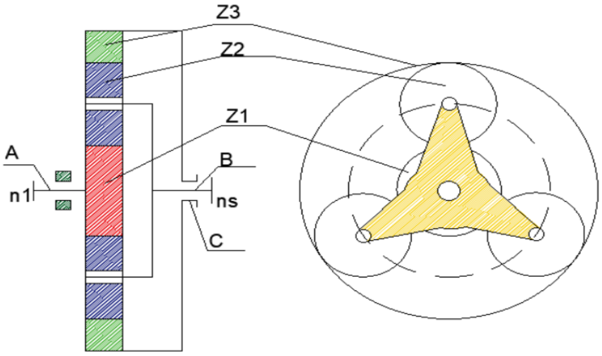


*Fig. 3: Shafts of the planetary transmission*

Input shaft is shaft A

$$P_A = \omega_A * T_A, \text{ [W]}$$

Angular velocity of the shafts:

$$\omega_A = n_A * \frac{\pi}{30} \text{ [s}^{-1}\text{]}$$
$$\omega_B = n_B * \frac{\pi}{30} \text{ [s}^{-1}\text{]}$$
$$\omega_C = n_C * \frac{\pi}{30} \text{ [s}^{-1}\text{]}$$

Torque of the input shaft A:

$$T_A = \frac{P_A}{\omega_A}$$

Torque of shaft B:

$$T_B = -(T_A + T_C) = -(1 - i_0 * \eta_0) * T_A$$

Torque of shaft C:

$$T_C = -i_0 * \eta_0 * T_A \text{ [Nm]}$$

Due to the division of the power on $P_k$ and $P_z$, the power on the shaft A is:

$$P_{ZA} = \omega_{AS} * T_A$$
$$P_{ZA} = (\omega_{A0} - \omega_{s0}) * T_A$$
$$P_{KA} = \omega_{S0} * T_A \text{ [W]}$$

The powers on shaft B are:

$$P_{ZB} = \omega_{BS} * T_B = (\omega_{B0} - \omega_{s0}) * T_B$$
$$P_{KB} = \omega_{S0} * T_B$$

The powers on shaft C are:

$$P_{ZC} = \omega_{CS} * T_C = (\omega_{C0} - \omega_{s0}) * T_C$$
$$P_{KC} = \omega_{S0} * T_C$$

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

**Calculation of the orientation module of the gear pair $z_1 - z_2$**

Module of the gear pairs $z_1$-$z_2$ is calculated using the formula [6]:

$$m \geq \sqrt[3]{\frac{2 * T_1}{\lambda * z_1 * \sigma_{FP}} * Y_{Fa} * Y_{Sa} * K_{F\alpha} * K_{F\beta} * K_v * K_a}$$

$Y_{Fa}$ - tooth shape factor, $Y_{Sa}$ – stress correction factor, $K_{F\alpha}$ - load distribution factor in the frontal cross-section for stress at the tooth root, $K_{F\beta}$ - load distribution factor along the length of the tooth for stress at the tooth root, $K_v$ - distribution factor of the peripheral force along the engagement of central gears with planetary gears, $K_a$ - factor for uniform loading of the machine in continuous operation with the electric motor, $\lambda$ - tooth width factor and $\sigma_{FP}$ - root bending stress [6]. As a material for the manufacture of the central gear with external teeth, it is choiced induction-hardened steel.

Because there are three planetary gears in the planetary transmission, the total power is divided into 3 parts:

$$P_1 = \frac{P_{ZA}}{N}$$

The torque at the point of contact between gear 1 and one of the planetary gears 2 will be:

$$T_1 = \frac{P_1}{\omega_1}$$

**Calculation of the orientation module of the gear pair $z_2 - z_3$**

Because this planetary transmission is of the 1AI type, all gears need to have the same module to achieve mutual engagement. It was necessary to calculate the orientation module of the gear pair, and then a module that meets the requirements for both gear pairs was selected.

We follow the same procedure as with the gear pair $z_1 - z_2$, so the next step is [6]:

$$m \geq \sqrt[3]{\frac{2 * T_2}{\lambda * z_2 * \sigma_{FP}} * Y_F * Y_\varepsilon * K_{H\alpha} * K_{H\beta} * K_v * K_I}$$

$K_{H\alpha}$ – transverse load factor for contact stress, $K_{H\beta}$ – face load factor contact stress

Because there are 3 planetary gears in the planetary transmission, the total power is divided into 3 parts:

$$P_2 = \frac{P_{ZC}}{N}$$

The torque at the point of contact between gear 3 and one of the planetary gears 2 will be:

$$T_2 = \frac{P_2}{\omega_2}$$

Where the angular velocity of the planetary gear is $\omega_2$, and it is:

$$\omega_2 = \frac{\pi * n_2}{30}$$

**Dimensions of the gears**

The width of the teeth is determined from the expression for the tooth width factor.

The width of the teeth is $\lambda = b/m$

The center distance between the two gear pairs is equal and is:

$a_{12} = a_{23}$, $a_{12} = \frac{(z_1+z_2)*m/2}{2}$

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

Diameters of gear 1 (central gear with external teeth)
Reference diameter:$d_1 = m * z_1$, Tip diameter:$d_{a1} = d_1 + 2m$, Root diameter:$d_{f1} = d_1 - 2{,}5m$
Diameters of gear 2 (planetary gears)
Reference diameter: $d_2 = m * z_2$, Tip diameter:$d_{a2} = d_2 + 2m$, Root diameter:$d_{f2} = d_2 - 2{,}5m$
Diameters of gear 3 (central gear with internal teeth)
Reference diameter:$d_3 = m * z_3$, Tip diameter:$d_{a3} = d_3 + 2m$, Root diameter:$d_{f3} = d_3 - 2{,}5m$

**Inspection of teeth root bending stress**

Gear pair $\boldsymbol{z_1 - z_2}$
The teeth root bending stress is calculated according to the equation [7]:

$$\sigma_{F1} = \frac{F_{tw1}}{b * m} * Y_{Fa} * Y_{Sa} * Y_B * K_{F\alpha} * K_{F\beta} * K_v * K_a$$

The force $F_{tw1}$ is:

$$F_{tw1} = \frac{2 * T_1}{dw_1}$$

Gear pair $\boldsymbol{z_2 - z_3}$

$$\sigma_{F2} = \frac{F_{tw2}}{b * m} * Y_{Fa} * Y_{Sa} * Y_B * K_{F\alpha} * K_{F\beta} * K_v * K_a$$

The force $F_{tw2}$ is:

$$F_{tw2} = \frac{2 * T_2}{dw_2}$$

## RESULTS AND DISCUSSION

For the given values $z_1$=25, $z_2$=85 and $z_3$=195, P=4.000[W] the planetary reducer transmission ratios were first calculated. Then, bending stresses in the roots of the teeth of the gear pair $z_1$-$z_2$ were calculated for different numbers of revolutions of the input shaft, for a number of revolutions from 500 to 6,000 revolutions ($z_2$ and $z_3$ are not changed). Obtained values for $z_1$=25 are shown in table 2. Based on the values from table 2 is drawn the diagram presented in the Figure 4. This curve actually determines the gear pair $z_1$-$z_2$ bending stress change depending on the number of revolutions of the input shaft $n_1$. The equation of this curve is y=58716/x and it was calculated using approximation method.

***Table 2: Bending stress of teeth of gear pair $z_1$-$z_2$***

| $Z_1$ | Number of revolutions per minute of the Input Shaft of Planetary Reducer | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **500** | **750** | **1000** | **1500** | **2000** | **2500** | **3000** | **3500** | **4000** | **4500** | **5000** | **5500** | **6000** |
| **21** | 117.57 | 78.38 | 58.78 | 39.19 | 29.39 | 23.52 | 19.59 | 16.80 | 14.70 | 13.06 | 11.77 | 10.69 | 9.8 |
| **23** | 104.75 | 69.83 | 52.38 | 34.92 | 26.19 | 20.95 | 17.46 | 14.96 | 13.09 | 11.64 | 10.48 | 9.52 | 8.73 |
| **25** | 93.99 | 62.66 | 46.99 | 31.33 | 23.50 | 18.80 | 15.66 | 13.43 | 11.75 | 10.44 | 9.4 | 8.54 | 7.83 |
| **27** | 85.89 | 57.26 | 42.94 | 28.63 | 21.47 | 17.18 | 14.31 | 12.27 | 10.74 | 9.54 | 8.59 | 7.81 | 7.16 |
| **29** | 78.91 | 52.6 | 39.45 | 26.3 | 19.73 | 15.78 | 13.15 | 11.27 | 9.86 | 8.77 | 7.89 | 7.17 | 6.58 |

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

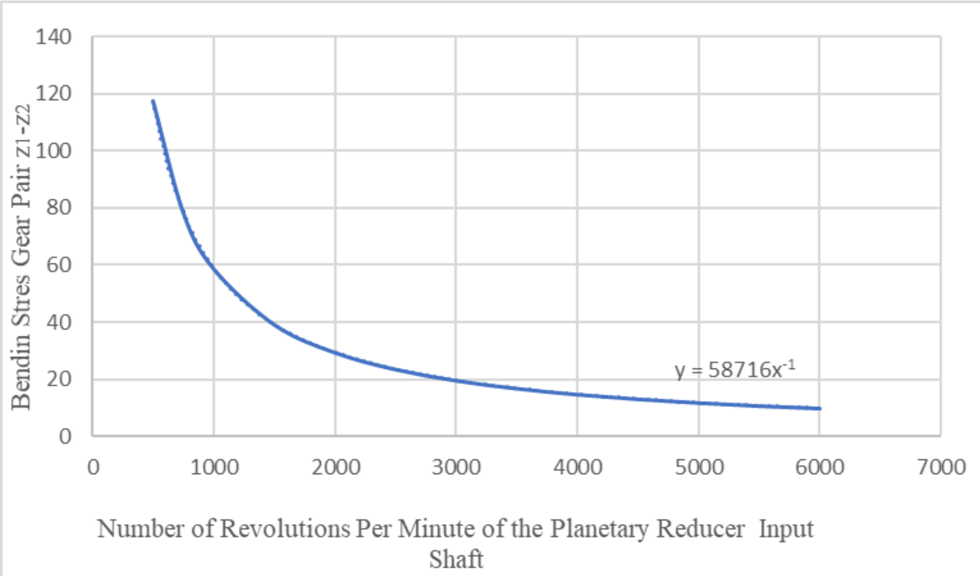


***Fig. 4: Bending Stress of the Gear Pair ($z_1$-$z_2$) Teeth for Different Number of Revolutions of the Planetary Input Shaft***

Next calculations were made for the cases when the number of teeth on the driving shaft is changed from $z_1$=21, $z_1$=23, $z_1$=27 to $z_1$=29. For each of these cases the bending stresses were calculated for different input shaft numbers of revolutions from 500 to 6000 revolutions per minute.

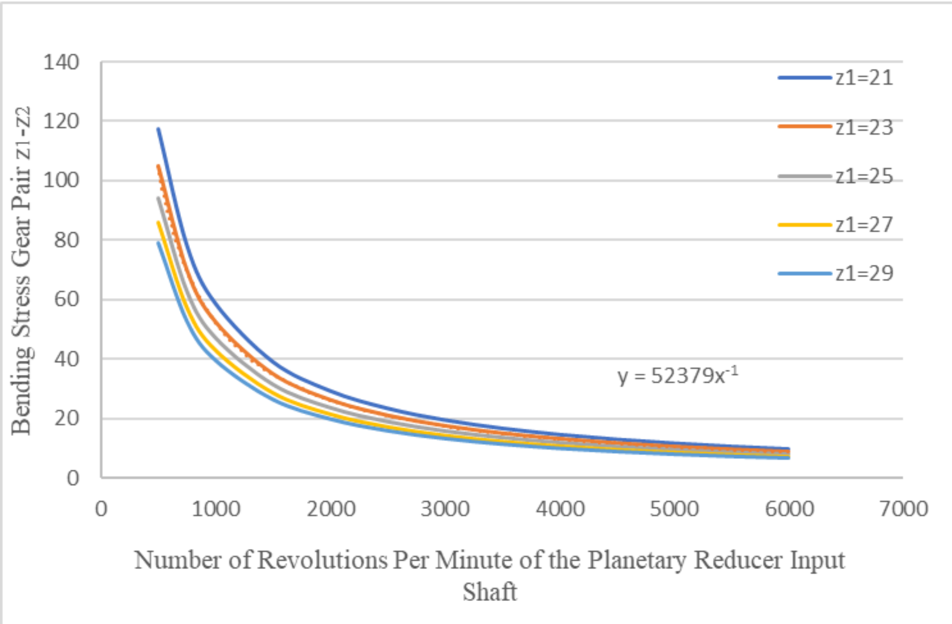


***Fig. 5: Bending Stress of the Gear Pair ($z_1$-$z_2$) Teeth for Different Number of Revolutions of the Planetary Input Shaft for Different Values of $z_1$***

iJETRM
International Journal of Engineering Technology Research & Management
www.ijetrm.com

## CONCLUSION

With a constant number of teeth in the planetary reducer (constant transmission ratio) and constant power of the input shaft, with an increase in the number of revolutions of the input shaft, the bending stress in the roots of the teeth of the gear pair $z_1$-$z_2$ decreases. The degree of bending stress reduction for a lower number of revolutions (up to 1000 revolutions per minute) is the greatest. With the further increase in the number of revolutions of the input shaft, the reduction of the bending stress on the teeth continues but with a reduced intensity. For the number of revolutions greater than 4000 revolutions per minute the reduction of the bending stress is insignificant. This fact can be used in cases where it is necessary for a certain planetary reducer to reduce the bending stress at the roots of the teeth.

From the Diagrams in Figure 5, it can be seen that the reducing the bending stress in the teeth ($z_1$=21,23,27,29) also applies upon changing the number of teeth of the drive gear. By increasing the number of teeth of the drive gear, the bending stress in the teeth for the same number of revolutions of the input shaft decreases and vice versa.

## REFERENCES

[1] Meriam J., Kraige L., Bolton J, Wiley (2018) *Engineering Mechanics*, Volume 2, Dynamics
[2] Stamboliev D., Ss.Cyril and Methodius University in Skopje, Macedonia ((1976) Prenosnici na vozilata
[3] Petrovic T, Ivanov T and Milosevic M, , Forsch Ingenieures (2009), *A new structure of combined gear trains with high transmission ratios*
[4] Arnaudov K., Karaivanov D., CRC Press, Taylor & Francis Group (2018), Planetary Gear Trains
[5] Simeonov S., Milev S., Goce Delcev University Stip (2019), *Praktikum po Masinski elementi*
[6] Simeonov S., Goce Delcev University Stip (2017), *Masinski elementi*
[7] Budinas R., Nissbet J., Mc Graw Hill Education (2020) *Shigley's Mechanical Engineering Design*
[8] Jiang W., Wiley (2019). *Analyses and Design of Mechanical Elements.*