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RP-83: FORMULATION OF SOLUTIONS OF STANDARD BIQUADRATIC CONGRUENCE OF EVEN COMPOSITE MODULUS-A MULTIPLE OF AN ODD INTEGER

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ABSTRACT
In this paper, a standard solvable bi-quadratic congruence of even composite modulus-a multiple of an odd positive integer, is considered for formulation of its solutions. It is discussed and its solutions are formulated. Now, it becomes possible to find all the solutions directly and orally. No discussion is found in the earlier literature of mathematics. A formulation is established first time for the solutions of the congruence under consideration. This is the merit of the paper.

Key-words: Bi-quadratic congruence, Binomial expansion, Formulation.

INTRODUCTION

A bi-quadratic congruence is a topic of congruence in Number Theory. No detailed discussion is found in the literature of mathematics. Only quadratic congruence is discussed prominently and studied. Euler, Gauss and Fermat had given much attention to the congruence of standard quadratic congruence. The author found a vast gap in the literature for research and has started his journey to study the standard cubic congruence and also the standard bi-quadratic congruence for formulation. Here the author considered a very special type of standard bi-quadratic congruence of even composite modulus for its formulation.

PROBLEM-STATEMENT

Here the problem is:
"To formulate the standard solvable bi-quadratic congruence of even composite modulus of the type:

$x^4 \equiv a^4 \ (mod\ 2^m.b); m \geq 3, b\ an\ odd\ positive\ integer,$ in two cases:

Case-I: If a is odd positive integer;

Case-II: If a is even positive integer.

LITERATURE-REVIEW

Nothing is seen in the literature of mathematics for the said congruence. Only a definition of bi-quadratic residue is mentioned. Also the solvability condition is not found. It is said that if $x^4 \equiv a \ (mod\ p)$ is solvable, then a is called the bi-quadratic residue of p **[7].** Earlier mathematicians approached less to the topic. The author first time tried his best to formulate some standard bi-quadratic congruence successfully such as:

$x^4 \equiv a^4 \ (mod\ 4.p^n)$

& $x^4 \equiv a^4 \ (mod\ 8.p^n);$

$x^4 \equiv a^4 \ (mod\ 2^m.p^n);$

$x^4 \equiv a^4 \ (mod\ 4^n.b).$

$x^4 \equiv a^4 \ (mod\ 2^m).$

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p being a positive prime integer; n any positive integer **[1], [2], [3], [4], [5], [6].**

In continuation of the above research, the author considered the next research paper:

$x^4 \equiv a^4 \ (mod\ 2^m.b); m \geq 3, b$ *an odd positive integer,* for formulation and studied.

NEED OF RESEARCH

No formulation of the said congruence under consideration is found in the earlier literature of mathematics. No existed method is also found.

It is very difficult to find the solutions of the congruence. To get rid of such difficulty, the author considered the congruence for study and tried his best to formulate and presented his sincere efforts in this paper. To have a formula for solutions is the need of the research.

ANALYSIS & RESULT

Case-I: Let a≠1 be an odd positive integer.

Consider the said congruence: $x^4 \equiv a^4 \ (mod\ 2^m.b); m \geq 3.$

Let us consider that $x \equiv (2^{m-2}.b.k \pm a) \ (mod\ 2^m.b).; k = 0, 1, 2, \ldots\ldots\ldots\ldots$

Then,

$$x^4 \equiv (2^{m-2}.b.k \pm a)^4 (mod\ 2^m.b).$$

$$\equiv (2^{m-2}.b.k)^4 \pm 4.(2^{m-2}.b.k)^3.a + \frac{4.3}{1.2}(2^{m-2}.b.k)^2.a^2 \pm \frac{4.3.2}{1.2.3}(2^{m-2}.b.k)^1.a^3 + a^4$$

$$=\equiv a^4 + 2^m.b.k(t) \ (mod\ 2^m.b).; t \text{ is a positive integer.}$$

$$\equiv a^4 (mod\ 2^m.b).$$

Therefore, $x \equiv (2^{m-2}.b.k \pm a)$ (mod $2^m.b$) are the solutions of the said congruence.

For $k = 4,$ *the solution formla becomes* $x \equiv (2^{m-2}.b.4 \pm a) \ (mod\ 2^m.b)$

$$\equiv (2^m.b \pm a) \ (mod\ 2^m.b)$$

$\equiv \pm a \ (mod\ 2^m.b)$, which is the same solution as for $k = 0.$

For $k = 5 = 4 + 1 = 2^2 + 1,$ then the solutions formula becomes

$$x \equiv 2^{m-2}.b.(2^2 + 1) \pm a \ (mod\ 2^m.b)$$

$$\equiv \{2^{m-2}.b.2^2 + 2^{m-2}.b.1\} \pm a \ \ (mod\ 2^m.b)$$

$$\equiv \{2^m.b + 2^{m-2}.b\} \pm a) \ (mod\ 2^m.b)$$

$$\equiv (2^{m-2}.b \pm a) \ (mod\ 2^m.b)$$

Which is the same solution as for k=1.

Similarly for $k = 6,$ 7, the solutions are the same as for $k = 2, 3, \ldots\ldots$

Thus, all the solutions are given by $x \equiv (2^{m-2}.b.k \pm a) \ (mod\ 2^m.b); k = 0, 1, 2, 3, 4.$

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Therefore, the congruence has exactly eight solutions.

Case-II:

Let a be an even positive integer.

Consider the said congruence: $x^4 \equiv a^4 \ (mod\ 2^m.b); m \geq 3.$

Let us consider that $x = (2^{m-3}.b.k \pm a); k = 0, 1, 2, \ldots\ldots\ldots\ldots$

Then,

$$x^4 = (2^{m-3}.b.k \pm a)^4$$

$$= (2^{m-3}.b.k)^4 \pm 4.(2^{m-3}.b.k)^3.a + \frac{4.3}{1.2}(2^{m-3}.b.k)^2.a^2 \pm \frac{4.3.2}{1.2.3}(2^{m-3}.b.k)^1.a^3 + a^4$$

$$= a^4 + 2^m.b.k(t); t \ a \ positive \ integer.$$

$$\equiv a^4 (mod\ 2^m.b).$$

Therefore, $x \equiv (2^{m-3}.b.k \pm a)$ (mod $2^m.b$) are the solutions of the said congruence.

For $k = 8$, *the solution formla becomes* $x = (2^{m-3}.b.8 \pm a)\ (mod\ 2^m.b)$

$\equiv (2^m.b \pm a)\ (mod\ 2^m.b)$

$\equiv \pm a\ (mod\ 2^m.b)$, which is the same solution as for $k = 0$.

For $k = 9 = 8 + 1 = 2^3 + 1$, then the solution formula becomes

$$x \equiv 2^{m-3}.b.(2^3 + 1) \pm a\ (mod\ 2^m.b)$$

$$\equiv \{2^{m-3}.b.2^3 + 2^{m-3}.b.1\} \pm a \ \ (mod\ 2^m.b)$$

$$\equiv \{2^m.b + 2^{m-3}.b\} \pm a)\ (mod\ 2^m.b)$$

$$\equiv (2^{m-3}.b \pm a)\ (mod\ 2^m.b)$$

Which is the same solution as for k=1.

Similarly for $k = 10$, 11, the solutions are the same as for $k = 2, 3, \ldots\ldots$

Thus, all the solutions are given by $x \equiv (2^{m-3}.b.k \pm a)\ (mod\ 2^m.b); k = 0, 1, 2, 3 \ldots\ldots 7.$

Therefore, the congruence has exactly sixteen solutions.

ILLUSTRATIONS

Consider the congruence $x^4 \equiv 1\ (mod\ 48)$ *for* $a = 1$.

It can be written as $x^4 \equiv 1^4 (mod\ 2^4.3)$.

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It is of the type $x^4 \equiv a^4 (mod\ 2^m.b)$ $with\ a = 1,\ m = 4, b = 3.$

The solutions are given by

$$x \equiv (2^{m-3}.b.k \pm a)\ (mod\ 2^m.b).$$

$$\equiv (2^{4-3}.3.k \pm 1)\ (mod\ 2^4.3)$$

$$\equiv (6k \pm 1)\ (mod\ 2^4.3); k = 0, 1, 2, 3.$$

$$\equiv \pm 1; 6 \pm 1; 12 \pm 1,; 18 \pm 1; 24 \pm 1; 30 \pm 1; 36 \pm 1;\ 42 \pm 1 (mod\ 48)$$

$$\equiv 1, 47; 5, 7; 11, 13; 17, 19; 23, 25; 29, 31; 35, 37; 41, 43\ (mod\ 48).$$

Every solution is tested and verified true. There are sixteen solutions.

Consider the congruence $x^4 \equiv 16\ (mod\ 48).$

It can be written as $x^4 \equiv 16\ (mod\ 16.3)\ i.e. x^4 \equiv 2^4\ (mod\ 2^4.3)$

It is of the type $x^4 \equiv\ a^4\ (mod\ 2^m.b)$ $with\ b = 3, m = 4.$

Thus the solutions are given by

$x \equiv 2^{m-3}bk \pm a\ (mod\ 2^m b)$ for $k = 0, 1, 2, 3, \ldots\ldots 7.$

$$\equiv 2^{4-3}.3k \pm 2\ \ (mod\ 2^4.3)$$

$$\equiv 6k \pm 2\ (mod\ 48); k = 0, 1, 2, 3\ \ldots\ldots 7.$$

$$\equiv \pm 2, 6 \pm 2, 12 \pm 2, 18 \pm 2; 24 \pm 2; 30 \pm 2; 36 \pm 2; 42 \pm 2\ (mod\ 48)$$

$$\equiv 2, 46; 4, 8; 10, 14; 16, 20; 22, 26; 28, 32; 34, 38; 40, 44\ (mod\ 48)$$

Therefore, the solutions are

$x \equiv 2, 46, 4, 8, 10, 14, 16, 20, 22, 26, 28, 32, 34, 38, 40, 44\ (mod\ 48).$

Consider the congruence $x^4 = 81\ (mod\ 160).$

It can be written as $x^4 \equiv\ 3^4 (mod\ 2^5.5).$

It is of the type $x^4 \equiv a^4 (mod\ 2^m.b)$ $with\ a = 3, an\ odd\ integer,\ m = 5, b = 5.$

So, it has exactly eight solutions.

The solutions are given by

$$x \equiv (2^{m\ -2}.b.k \pm a)\ (mod\ 2^m.b).$$

$$\equiv (2^{5-2}.5.k \pm 3)\ (mod\ 2^5.5)$$

$$\equiv (40k \pm 3)\ (mod\ 2^5.5); k = 0, 1, 2, 3.$$

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$\equiv \pm 3; 40 \pm 3; 80 \pm 3; 120 \pm 3 (mod\ 160)$

$\equiv 1, 157; 37, 43; 77, 83; 117, 123\ (mod\ 160).$

It can be written as: $x^4 \equiv 256\ (mod\ 32.17)$

$i.e.\ x^4 \equiv 4^4\ (mod\ 2^5.17)$

It is of the type: $x^4 \equiv a^4\ (mod\ 2^m.b)$ with b=17, m = 5, a = 4, an even integer.

So, it has exactly sixteen solutions.

The solutions are:

$x \equiv 2^{m-3}.b.k \pm a\ (mod\ 2^m.b); k = 0, 1, 2, 3, 4, 5, 6, 7.$

$x \equiv 4.17k \pm a\ (mod\ 2^m.b)$

$\equiv 68k \pm 2\ (mod\ 2^5.17)$

$\equiv 68k \pm 2\ (mod\ 544)$

$\equiv 0 \pm 2; 68 \pm 2; 136 \pm 2; 204 \pm 2; 272 \pm 2; 340 \pm 2; 408 \pm 2; 476 \pm 2$ (mod 544)

$\equiv 2, 542; 66, 70; 134, 138; 202, 206; 270, 274; 338, 342; 406, 410; 474, 478$ (mod 544)

Consider the congruence $x^4 \equiv 49\ (mod\ 288).$

It can be written as: $x^4 \equiv 49 + 576 = 625 = 5^4\ (mod\ 32.9)$

$i.e.\ x^4 \equiv 5^4\ (mod\ 2^5.9)$

It is of the type: $x^4 \equiv a^4\ (mod\ 2^m.b)$ with b=9, m=5, a=5, an odd positive integer.

The solutions are given by

$x \equiv 2^{m-2}.b.k \pm a\ (mod\ 2^m.b); k = 0, 1, 2, 3.$

$\equiv 2^{5-2}.9.k \pm 5\ (mod\ 2^5.9); k = 0, 1, 2, 3.$

$\equiv 72k \pm 5\ (mod\ 288)$

$\equiv 0 \pm 5; 72 \pm 5; 144 \pm 5; 216 \pm 5\ (mod\ 288)$

≡4, 283; 67, 77; 139, 149; 211, 221 (mod 288).

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CONCLUSION

Therefore, it can be concluded that the standard solvable bi-quadratic congruence under consideration $x^4 \equiv a^4 (mod\ 2^m.b); m \geq 4; a \neq 1, b\ an\ odd\ positive\ integer$, has exactly eight incongruent solutions which are

given by: $x \equiv (2^{m-2}.b.k \pm a)\ (mod\ 2^m.b); k = 0, 1, 2, 3$,

$if\ a\ is\ an\ odd\ positive\ integer.$

Also, the standard solvable bi-quadratic congruence under consideration

$x^4 \equiv a^4 (mod\ 2^m); m \geq 4$ has exactly sixteen incongruent solutions which are

given by: $x \equiv (2^{m-3}.b.k \pm a)\ (mod\ 2^m.b); k = 0, 1, 2, 3, 4, 5, 6, 7,$

$if\ a\ is\ an\ even\ positive\ integer.$

MERIT OF PAPER

In this paper, the standard solvable bi-quadratic congruence is studied and formulated. It now becomes easy to find all the solutions directly. Thus, formulation of the solutions of the congruence is the merit of the paper.

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