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FLEXURAL INVESTIGATION OF THICK BEAM USING REFINED SHEAR DEFORMATION THEORY

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ABSTRACT
The structural behaviour of thick beams has garnered significant attention due to their wide applications in various engineering and construction fields. This study presents a comprehensive investigation into the flexural behaviour of thick beams utilizing the refined shear deformation theory. The refined shear deformation theory offers a refined approach to capture the accurate shear deformation effects that are prominent in thick beams, which are often inadequately addressed by classical beam theories.
In this research, a detailed theoretical framework of the refined shear deformation theory is presented, highlighting its advantages in modelling thick beam behaviour. The governing equations are derived considering both the shear deformation and transverse normal strain effects, offering a more accurate representation of the complex stress and strain distributions that occur in thick beams subjected to bending.

Keywords:
Thick beam, trigonometric shear deformation, Flexural Investigation, equilibrium equations, displacement, stress

1. INTRODUCTION
Investigation is a need to study the complex real-life problems in the field of construction now a days. Additionally, better comprehension of underlying conduct is needed for primary designing application. This inspires researchers to examine twisting, clasping, vibration, and so on of overlaid composite designs thinking about different perspectives. Many investigations have been led already to investigate the variety of structural actions to analyze and design the structural components.
The structural analysis of shear flexible structures heavily relies on the transverse shear deformation effect. In order to address the proper structural behavior, more sophisticated theories were developed as a result of the flexural analysis of thick beams. Since Bernoulli-presumptions Euler's are the basis of the elementary theory of beam (ETB) [1], We frequently employed the ETB, a model of axial forces and beam bending behavior, to investigate the behavior of bending components.
Later Timoshenko [2] developed the for the flexural behavior of moderately thick and thick beams, first order shear deformation theory (FSDT) is used. in which rotary inertia and shear deformation is taken into account. This theory requires a shear correction factor since it assumes that the transverse shear strain distribution is constant.
The Timoshenko's shear coefficient for beam flexural vibration is shown in Mindlin [3]. Higher order or more sophisticated shear deformation theories were created as a result of the correction factor in ETB and FSDT. Heyliger and Reddy [3] and Krishna Murty [4] Trigonometric and hyperbolic functions were used to create higher order shear deformation theories. As the shear stress free boundary condition is not met by the higher order shear deformation theory, Ghugal and Shimpi [5] addressed a review of the more sophisticated shear deformation theories for isotropic and anisotropic laminated beams and overcame this disadvantage. For the static and free vibration flexural analysis of thick beams, Ghugal and Sharma [6] devised the hyperbolic shear deformation theory. Both Ghugal and Dahake [7 For the flexural of thick beams that were simply supported and cantilevered, Dahake and Ghugal [8] used the revised shear deformation theory. For static bending and elastic buckling of P-FGM beams, Ghumare and Sayyad [9] established a novel fifth order shear and normal deformation theory. In order to demonstrate the efficacy of the theories and compare their results with those of other improved theories, analysis of the simply supported thick irregular shape beam with uniformly increasing load is taken into consideration in this study.

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2. DEVELOPMENT OF THEORY

The beam under consideration as shown in Fig.1 occupies in $0-x-y-z$ Cartesian coordinate system the region:

$$0 \le x \le L \; ; \quad 0 \le y \le b \; ; \quad -\frac{h}{2} \le z \le \frac{h}{2}$$

where *x*, *y*, *z* are Cartesian coordinates, *L* and *b* are the length and width of beam in the *x* and *y* directions respectively, and *h* is the thickness of the beam in the *z*-direction. The beam is made up of homogeneous, linearly elastic isotropic material.

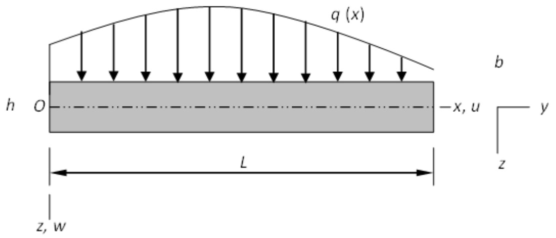

Fig.1 Beam under bending in x-z plane

2.1 The displacement field

The displacement field of the present beam theory is of the form:

$$u(x,z) = -z\frac{dw}{dx} + \frac{h}{\pi}\sin\frac{\pi z}{h}\phi(x)$$
$$w(x,z) = w(x) \tag{1}$$

Here, *u* is the axial displacement component in the *x* direction, and *w* is the transverse displacement in the *z* direction. The trigonometric function in terms of thickness coordinate in the displacement field of *u* is associated with the transverse shear stress distribution through the thickness of beam and the functions ϕ (*x*) is the unknown function associated with the shear slope /warping of the cross section of beam at neutral axis of beam.

Normal Strain:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{d^2 w}{dx^2} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{d\phi}{dx} \tag{2}$$

Shear Strain:

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{dw}{dx} = \cos\frac{\pi z}{h}\phi \tag{3}$$

Stress-strain relationships used are as follows

$$\sigma_x = E\varepsilon_x = -Ez\frac{d^2 w}{dx^2} + \frac{Eh}{\pi}\sin\frac{\pi z}{h}\frac{d\phi}{dx}$$
$$\tau_{zx} = G\gamma_{zx} = G\cos\frac{\pi z}{h}\phi \tag{4}$$

2.2 Governing equations and boundary conditions

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Using the expressions for strains and stresses (2) through (4) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the beam under consideration can be obtained. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x\delta\varepsilon_x+\tau_{zx}\delta\gamma_{zx}\right)dx\,dz-\int_{x=0}^{x=L}q(x)\,\delta w\,dx=0 \tag{5}$$

where the symbol δ denotes the variational operator. Employing Green's theorem in Eqn. (4) successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\,\frac{d^4w}{dx^4}-\frac{24}{\pi^3}\,EI\,\frac{d^3\phi}{dx^3}=q(x) \tag{6}$$

$$\frac{24}{\pi^3}\,EI\,\frac{d^3w}{dx^3}-\frac{6}{\pi^2}\,EI\,\frac{d^2\phi}{dx^2}+\frac{GA}{2}\phi=0 \tag{7}$$

The associated consistent natural boundary conditions obtained are of following form:

at the ends $x = 0$ and $x = L$ is of following form:

$$\text{Either}\quad V_x=EI\,\frac{d^3w}{dx^3}-\frac{24}{\pi^3}\,EI\,\frac{d^2\phi}{dx^2}=0\quad\text{or}\quad w\ \text{is prescribed} \tag{8}$$

$$\text{Either}\quad M_x=EI\,\frac{d^2w}{dx^2}-\frac{24}{\pi^3}\,EI\,\frac{d\phi}{dx}=0\quad\text{or}\quad \frac{dw}{dx}\ \text{is prescribed} \tag{9}$$

$$\text{Either}\quad M_s=EI\,\frac{24}{\pi^3}\frac{d^2w}{dx^2}-\frac{6}{\pi^2}\,EI\,\frac{d\phi}{dx}=0\quad\text{or}\quad \phi\ \text{is prescribed} \tag{10}$$

Thus the boundary value problem of the beam bending is given by the above variationally consistent governing differential equations and boundary conditions.

2.3 The General solution of governing equilibrium equations of the beam

The general solution for transverse displacement w(x) and warping function ϕ (x) is obtained using Eqns. (6) and (7) using method of solution of linear differential equations with constant coefficients. Integrating and rearranging the first governing Eqn. (6), we obtain the following equation

$$\frac{d^3w}{dx^3}=\frac{24}{\pi^3}\frac{d^2\phi}{dx^2}+\frac{Q(x)}{EI} \tag{11}$$

where Q(x) is the generalized shear force for beam and it is given by $Q(x)=\int^x q\,dx+C_1$

Now the second governing Eqn. (7) is rearranged in the following form:

$$\frac{d^3w}{dx^3}=\frac{\pi}{4}\frac{d^2\phi}{dx^2}-\beta\phi \tag{12}$$

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A single equation in terms of ϕ is now obtained using Eqns. (11) and (12) as:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q(x)}{\alpha EI} \tag{13}$$

where constants α , β and λ in Eqns. (12) and (13) are as follows

$$\alpha = \left(\frac{\pi}{4} - \frac{24}{\pi^3}\right),\ \beta = \left(\frac{\pi^3}{48}\frac{GA}{EI}\right) \quad \text{and } \lambda^2 = \frac{\beta}{\alpha}$$

The general solution of Eqn. (13) is as follows:

$$\phi(x) = C_2\cosh\lambda x + C_3\sinh\lambda x - \frac{Q(x)}{\beta EI} \tag{14}$$

The equation of transverse displacement w(x) is obtained by substituting the expression of ϕ (x) in Eqn. (12) and then integrating it thrice with respect to x. The general solution for w(x) is obtained as follows:

$$EI\,w(x) = \iiiint q\,dx\,dx\,dx\,dx + \frac{C_1x^3}{6} + \left(\frac{\pi}{4}\lambda^2 - \beta\right)\frac{EI}{\lambda^3}\left(C_2\sinh\lambda x + C_3\cosh\lambda x\right) + C_4\frac{x^2}{2} + C_5x + C_6 \tag{15}$$

where C_1, C_2, C_3, C_4, C_5 and C_6 are the arbitrary constants of integration and can be obtained by imposing natural (forced) and /or geometric or kinematical boundary / end conditions of beam.

3 ILLUSTRATIVE EXAMPLES

To prove the efficiency of the present theory, numerical examples are considered as follows,

3.1 Simply Supported beam (Symmetrical I Section) with uniformly varying load.

A simply supported beam has its length of 3 M and heigh of 1000 mm. where I-section beam consists of flange (500 mm X 250 mm) and web (150 mm X 500 mm). The beam is subjected to uniformly varying load of 20 KN/M on surface acting in the downward z direction with maximum intensity of load q = 20 kN/M.

The kinematic and static (forced) boundary conditions of beam are as follows:

$$\frac{\partial^2 w}{\partial x^2} = \frac{d\phi}{dx} = w = 0 \quad \ldots\text{at } x = 0, L \tag{3.1}$$

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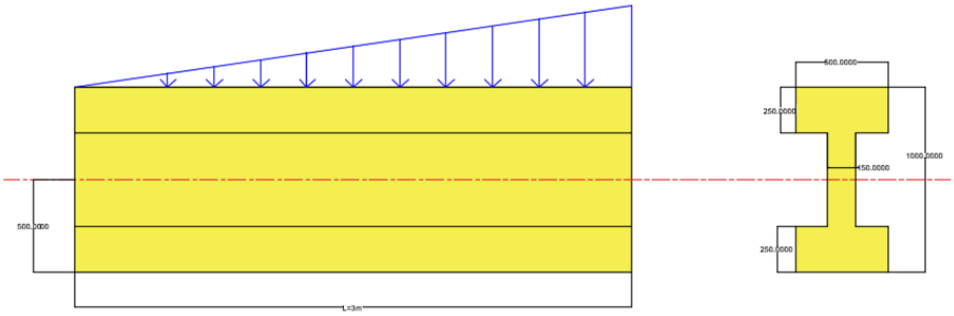

Fig 2. Symmetrical I-section beam with loading condition

Area of considered beam is $325\times10^{4}\ mm^{2}$, which is Consists of A1,A2 and A3.

Where ,

A1= Area of Fange= $125\times10^{3}\ mm^{2}$

A2= Area of Web $= 75\times10^{3}\ mm^{2}$

A3= Area of Flange= $125\times10^{3} mm^{2}$

Neutral axis i.e. $\overline{Y}$ of beam is $500\ mm$ from bottom

Moment of inertia of beam is i.e. $I=3.80\times10^{10}\ mm^{4}$

Young's modulus of beam is i.e., E=210 GPa

3.2 Analytical solution of Beam by using ETB method

The beam under consideration occupies in $0-x-y-z$ Cartesian coordinate system the region:

$$0\le x\le L\ ;\quad 0\le y\le b\ ;\quad -h/2\le z\le h/2 \tag{3.2.1}$$

where,

x, *y*, *z* = Cartesian coordinates, *L* = Length in *x* direction, *B* = Breadth in *y* directions and *h* = Thickness in the *z*-direction.

Based on a few kinematical and physical presumptions, the analytical formulation of a uniform thick beam is offered. Utilizing the virtual work concept, the variationally correct forms of differential equations and boundary conditions based on an assumed displacement field will be derived.

Boundary condition: - Simply supported beam with uniformly varying load.

$$\frac{\partial^{2}w}{\partial x^{2}}=\frac{d\phi}{dx}=w=0 \quad \text{……………... at } x = 0, L \tag{3.2.2}$$

Governing differential equation is

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$$EI\frac{d^4w}{dx^2} = q_0\left(\frac{x}{L}\right) \tag{3.2.3}$$

Integrating w.r.t. *x*, we get,

$$EI\frac{d^3w}{dx^3} = q_0\left(\frac{x^2}{2L}\right) + C_1 \tag{3.2.4}$$

Integrating w.r.t. *x*, we get,

$$EI\frac{d^2w}{dx^2} = q_0\left(\frac{x^3}{6L}\right) + C_1x + C_2 \tag{3.2.5}$$

Integrating w.r.t. *x*, we get,

$$EI\frac{dw}{dx} = q_0\left(\frac{x^4}{24L}\right) + C_1\frac{x^2}{2} + C_2x + C_3 \tag{3.2.6}$$

Integrating w.r.t. *x*, we get,

$$EIw = q_0\left(\frac{x^5}{120L}\right) + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4 \tag{3.2.7}$$

Now, applying boundary condition for equation (3.2.5)

$$\frac{d^2w}{dx^2} = 0.........@\,x = 0$$

$$C_2 = 0 \tag{3.2.8}$$

Now, applying boundary condition for equation (3.2.5)

$$\frac{d^2w}{dx^2} = 0.........@\,x = L$$

$$C_1 = -q_0\left(\frac{L}{6}\right) \tag{3.2.9}$$

Now, substitute the values of C1 and C2 in equation (3.2.5)

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$$\frac{d^2w}{dx^2} = \frac{q_0}{D}\left(\frac{x^3}{6L} - \frac{Lx}{6}\right)$$

Rearranging above equation as,

$$\frac{d^2w}{dx^2} = \frac{q_0L^2}{D}\left(\frac{x^3}{6L^3} - \frac{x}{6L}\right) \tag{3.2.10}$$

Now, applying boundary condition for equation (3.2.7), we get,

$$w = 0.........@\, x = 0$$

$$C_4 = 0 \tag{3.2.11}$$

Now, applying boundary condition for equation (3.2.7), we get,

$$w = 0.........@\, x = L$$

$$C_3 = q_0\left(\frac{7}{360}\right)L^3 \tag{3.2.12}$$

Now, substituting values of C1, C2, C3, C4 from equation (3.2.9), (3.2.8), (3.2.12), (3.2.11) respectively to equation (3.2.7), we get,

$$w = q_0\frac{x^5}{120L} - q_0\frac{x^3L}{36} + q_0\frac{7xL^3}{360}$$

$$w = \frac{q_0}{D}\left(\frac{x^5}{120L} - \frac{x^3L}{36} + \frac{7}{360}xL^3\right)$$

$$w = \frac{q_0L^4}{D}\left(\frac{x^5}{120L^5} - \frac{x^3}{36L^3} + \frac{7x}{360L}\right) \tag{3.2.13}$$

Now, Expression for axial displacement, *u*

$$u = -z\frac{dw}{dx} \tag{3.2.14}$$

$$\frac{dw}{dx} = \frac{q_0L^4}{EI}\left(\frac{5x^4}{120L^5} - \frac{x^3}{36L^3} + \frac{7x}{360L}\right)$$

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$$\frac{dw}{dx} = -z\frac{q_0 L^4 12}{Ebh^3}\left(\frac{x^4}{24L^5} - \frac{x^2}{12L^2} + \frac{7}{360}\right)$$ (3.2.15)

Substituting the equation (3.2.15) in equation (3.2.14), we get,

$$u = -\frac{z}{h}\frac{12q_0 L^2}{Ebh^2}\left(\frac{x^4}{24L^4} - \frac{x^2}{12L^2} + \frac{7}{360}\right)$$ (3.2.16)

Now, Expression for axial stress, σ_x

Differentiating equation (3.2.16), we get

$$\sigma_x = \frac{du}{dx} = -\frac{z}{h}\frac{12q_0 L^2}{Ebh^2}\left(\frac{3x^2}{6L^3} - \frac{x}{6L}\right)$$

$$\sigma_x = \frac{du}{dx} = -\frac{z}{h}\frac{12q_0 L}{Ebh^2}\left(\frac{x^2}{2L^2} - \frac{x}{6}\right)$$ (3.2.17)

Now, Expression for transverse shear stress τ_{zx}^{EE} obtained from equilibrium equation $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$
(3.2.18)

The expression of obtained for transverse shear stress for this loading case is as follows:
Differentiating equation (3.2.17) w.r.t. *x*, we get,

$$\frac{\partial \sigma_x}{\partial x} = -\frac{z}{h}\frac{12q_0 L^2}{Ebh^2}\left(\frac{3x^2}{6L^3} - \frac{1}{6L}\right)$$

$$\frac{\partial \sigma_x}{\partial x} = -\frac{z}{h}\frac{12q_0 L}{Ebh^2}\left(\frac{x^2}{2L^2} - \frac{1}{6}\right)$$ (3.2.19)

Integrating equation (3.2.19) w.r.t. z, we get,

$$\tau_{zx} = -\int \frac{\partial \sigma_x}{\partial x} dz + k_1$$ (3.2.20)

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$$\tau_{zx} = -\int\left(-\frac{z^2}{h}\frac{12q_0L}{Ebh^2}\left(\frac{x^2}{2L^2}-\frac{1}{6}\right)\right)dz + k_1$$

$$\tau_{zx} = \frac{z^2}{2h}\frac{12q_0L}{Ebh^2}\left(\frac{x^2}{2L^2}-\frac{1}{6}\right)+k_1 \qquad (3.2.21)$$

Applying boundary condition as shear stress is zero at top and bottom of beam, i.e.

$\tau_{zx} = 0$ at $z = \frac{h}{2}$

$$0 = \frac{h^2}{8h}\frac{12q_0L}{Ebh^2}\left(\frac{x}{L}-\frac{x^2}{2L^2}-\frac{1}{6}\right)+k_1$$

$$k_1 = -\frac{h^2}{8h}\frac{12q_0L}{Ebh^2}\left(\frac{x}{L}-\frac{x^2}{2L^2}-\frac{1}{6}\right) \qquad (3.2.22)$$

Substituting value of k1 in equation (3.2.21), we get,

$$\tau_{zx} = \left(\frac{z^2}{2h}-\frac{h^2}{8h}\right)\frac{12q_0L}{Ebh^2}\left(\frac{x^2}{2L^2}-\frac{1}{6}\right)$$

$$\tau_{zx} = \left(4\frac{z^2}{h^2}-1\right)\frac{12q_0L}{8Ebh}\left(\frac{x^2}{2L^2}-\frac{1}{6}\right) \qquad (3.2.23)$$

4. NUMERICAL RESULT

4.1 Numerical Results for simply supported Symmetrical I- Section beam with uniformly varying load.

4.1.1 The results for transverse deflection **(w)**

The results for transverse deflection (w) are obtained from equation (3.2.13), equation (3.3.22), equation (3.4.29), equation (3.5.3), equation (3.6.3) and equation (3.7.3) of the ETB method, FSDT method, HSDT method, TSDT method, HPSDT method and V order method respectively in the following Table 1.

TABLE 1. Transverse Deflection (w) of the simply supported Symmetrical I-Section Beam Subjected to uniformly varying load

Source	Model	$\bar{w}$
Present	V Order	3.001143
Dahake and Ghugal	TSDT	3.015268
Ghugal and Sharma	HPSDT	3.013540
Krishna Murty	HSDT	3.014669

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Timoshenko	FSDT	2.687998
Bernoulli-Euler	ETB	1.169590

4.1.2 The results for axial displacement (u)

The results for axial displacement (u) are obtained from equation (3.2.16), equation (3.3.24), equation (3.4.34), equation (3.5.8), equation (3.6.8) and equation (3.7.8) of the ETB method, FSDT method, HSDT method, TSDT method, HPSDT method and V order method respectively in the Table 2.

Results in Table 2. Shows the Variation of axial displacement ($\bar{u}$) through the thickness of simply supported Symmetrical I-Section beam from top $+h/2$ to bottom $-h/2$ when subjected to load of 20 KN/M

TABLE 2. axial displacement ($\bar{u}$) through thickness of the Beam from $+h/2$ to bottom $-h/2$

	ETB	FSDT	HSDT	TSDT	HPSDT	V ORDER
0.5	-0.34667	-0.32255	-0.925236299	-0.923561322	-0.54361	-0.9546
0.48	-0.3328	-0.30964	-0.886119324	-0.88444404	-0.5054	-0.91544
0.46	-0.31893	-0.29674	-0.847260412	-0.845593064	-0.46921	-0.8764
0.44	-0.30507	-0.28384	-0.808648812	-0.806997298	-0.43496	-0.83747
0.42	-0.2912	-0.27094	-0.770273771	-0.768645645	-0.40255	-0.79865
0.4	-0.27733	-0.25804	-0.732124536	-0.730527009	-0.3719	-0.75994
0.38	-0.26347	-0.24514	-0.694190354	-0.692630295	-0.34293	-0.72133
0.36	-0.2496	-0.23223	-0.656460473	-0.654944406	-0.31556	-0.68281
0.34	-0.23573	-0.21933	-0.61892414	-0.617458246	-0.2897	-0.64438
0.32	-0.22187	-0.20643	-0.581570602	-0.580160719	-0.26527	-0.60604
0.3	-0.208	-0.19353	-0.544389108	-0.543040729	-0.24218	-0.56777
0.28	-0.19413	-0.18063	-0.507368903	-0.50608718	-0.22035	-0.52958
0.26	-0.18027	-0.15482	-0.470499236	-0.469288976	-0.1997	-0.49146
0.24	-0.55467	-0.51607	-1.445897844	-1.442116736	-0.60047	-1.51136
0.22	-0.50844	-0.47307	-1.323895009	-1.320380728	-0.53863	-1.38472
0.2	-0.46222	-0.43006	-1.202286439	-1.199051575	-0.47986	-1.25825
0.18	-0.416	-0.38706	-1.081036291	-1.078092292	-0.4239	-1.13195
0.16	-0.36978	-0.34405	-0.960108724	-0.95746589	-0.37045	-1.0058
0.14	-0.32356	-0.30104	-0.839467894	-0.837135383	-0.31925	-0.87979
0.12	-0.27733	-0.25804	-0.71907796	-0.717063785	-0.27	-0.75389

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0.1	-0.23111	-0.21503	-0.59890308	-0.597214108	-0.22244	-0.62809
0.08	-0.18489	-0.17202	-0.47890741	-0.477549365	-0.17627	-0.50237
0.06	-0.13867	-0.12902	-0.35905511	-0.35803257	-0.13122	-0.37672
0.04	-0.09244	-0.08601	-0.239310336	-0.238626735	-0.08701	-0.25112
0.02	-0.04622	-0.04301	-0.119637247	-0.119294874	-0.04337	-0.12555
0	0	0	0	0	0	0
-0.02	0.046222	0.012902	0.119637247	0.119294874	0.043367	0.125551
-0.04	0.092444	0.086012	0.239310336	0.238626735	0.087015	0.251118
-0.06	0.138667	0.129019	0.35905511	0.35803257	0.131222	0.376719
-0.08	0.184889	0.172025	0.47890741	0.477549365	0.176269	0.502369
-0.1	0.231111	0.215031	0.59890308	0.597214108	0.222436	0.628086
-0.12	0.277333	0.258037	0.71907796	0.717063785	0.270002	0.753886
-0.14	0.323556	0.301044	0.839467894	0.837135383	0.319248	0.879786
-0.16	0.369778	0.34405	0.960108724	0.95746589	0.370454	1.005803
-0.18	0.416	0.387056	1.081036291	1.078092292	0.423899	1.131952
-0.2	0.462222	0.430062	1.202286439	1.199051575	0.479864	1.258251
-0.22	0.508444	0.473068	1.323895009	1.320380728	0.538628	1.384716
-0.24	0.554667	0.516075	1.445897844	1.442116736	0.600471	1.511364
-0.26	0.180267	0.167724	0.470499236	0.469288976	0.199702	0.491464
-0.28	0.194133	0.180626	0.507368903	0.50608718	0.220355	0.529583
-0.3	0.208	0.193528	0.544389108	0.543040729	0.242183	0.567772
-0.32	0.221867	0.20643	0.581570602	0.580160719	0.265271	0.606035
-0.34	0.235733	0.219332	0.61892414	0.617458246	0.289703	0.644379
-0.36	0.2496	0.232234	0.656460473	0.654944406	0.315562	0.682807
-0.38	0.263467	0.245135	0.694190354	0.692630295	0.342934	0.721326
-0.4	0.277333	0.258037	0.732124536	0.730527009	0.3719	0.759939
-0.42	0.2912	0.270939	0.770273771	0.768645645	0.402547	0.798651
-0.44	0.305067	0.283841	0.808648812	0.806997298	0.434957	0.837469
-0.46	0.318933	0.296743	0.847260412	0.845593064	0.469215	0.876396
-0.48	0.3328	0.309645	0.886119324	0.88444404	0.505404	0.915438
-0.5	0.346667	0.322547	0.925236299	0.923561322	0.543609	0.9546

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4.1.3 The results for axial stress (σ_x)

The results for axial stress (σ_x) are obtained from equation (3.2.17), equation (3.3.25), equation (3.4.38), equation (3.5.11), equation (3.6.11) and equation (3.7.11) of the ETB method, FSDT method, HSDT method, TSDT method, HPSDT method and V order method respectively in the Table 3.

Results in Table 3. shows the variation of axial stress (σ_x) through the thickness of simply supported Symmetrical I-Section beam from top $+h/2$ to bottom $-h/2$ when subjected to load of 20 KN/M

TABLE 3. axial stress (σ_x) through thickness of the Beam from $+h/2$ to bottom $-h/2$

	ETB	FSDT	HSDT	TSDT	HPSDT	V ORDER
0.5	-0.000533333	-0.000533333	-0.000298456	-0.000301555	-0.001024885	-0.000240128
0.48	-0.000512	-0.000512	-0.000290532	-0.000293635	-0.001015243	-0.000232385
0.46	-0.000490667	-0.000490667	-0.000282116	-0.000285208	-0.001001761	-0.000224414
0.44	-0.000469333	-0.000469333	-0.00027323	-0.000276295	-0.000984601	-0.000216225
0.42	-0.000448	-0.000448	-0.000263893	-0.000266917	-0.000963921	-0.000207826
0.4	-0.000426667	-0.000426667	-0.000254125	-0.000257095	-0.000939882	-0.000199228
0.38	-0.000405333	-0.000405333	-0.000243949	-0.000246851	-0.000912644	-0.00019044
0.36	-0.000384	-0.000384	-0.000233383	-0.000236205	-0.000882366	-0.000181472
0.34	-0.000362667	-0.000362667	-0.000222448	-0.000225178	-0.000849209	-0.000172332
0.32	-0.000341333	-0.000341333	-0.000211165	-0.000213792	-0.000813333	-0.000163031
0.3	-0.00032	-0.00032	-0.000199555	-0.000202068	-0.000774897	-0.000153578
0.28	-0.000298667	-0.000298667	-0.000187637	-0.000190027	-0.000734061	-0.000143982
0.26	-0.000277333	-0.000277333	-0.000175432	-0.00017769	-0.000690986	-0.000134254
0.24	-0.000256	-0.000256	-0.000542348	-0.000549406	-0.002151916	-0.000413821
0.22	-0.000234667	-0.000234667	-0.000500031	-0.000506593	-0.001995074	-0.000380671
0.2	-0.000213333	-0.000213333	-0.000456963	-0.000463005	-0.001832366	-0.000347173
0.18	-0.000192	-0.000192	-0.000413213	-0.000418712	-0.001664326	-0.000313358
0.16	-0.000170667	-0.000170667	-0.000368848	-0.000373785	-0.001491487	-0.000279259
0.14	-0.000149333	-0.000149333	-0.000323937	-0.000328295	-0.001314383	-0.000244905
0.12	-0.000128	-0.000128	-0.000278547	-0.000282312	-0.001133546	-0.000210331
0.1	-0.000106667	-0.000106667	-0.000232749	-0.000235905	-0.000949509	-0.000175566
0.08	-8.53333E-05	-8.53333E-05	-0.000186609	-0.000189147	-0.000762807	-0.000140643
0.06	-0.000064	-0.000064	-0.000140195	-0.000142107	-0.000573971	-0.000105593
0.04	-4.26667E-05	-4.26667E-05	-9.35774E-05	-9.48554E-05	-0.000383536	-7.0448E-05

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0.02	-2.13333E-05	-2.13333E-05	-4.68228E-05	-4.74629E-05	-0.000192035	-3.52399E-05
0	0	0	0	0	0	0
-0.02	2.13333E-05	2.13333E-05	4.68228E-05	4.74629E-05	0.000192035	3.52399E-05
-0.04	4.26667E-05	4.26667E-05	9.35774E-05	9.48554E-05	0.000383536	7.0448E-05
-0.06	0.000064	0.000064	0.000140195	0.000142107	0.000573971	0.000105593
-0.08	8.53333E-05	8.53333E-05	0.000186609	0.000189147	0.000762807	0.000140643
-0.1	0.000106667	0.000106667	0.000232749	0.000235905	0.000949509	0.000175566
-0.12	0.000128	0.000128	0.000278547	0.000282312	0.001133546	0.000210331
-0.14	0.000149333	0.000149333	0.000323937	0.000328295	0.001314383	0.000244905
-0.16	0.000170667	0.000170667	0.000368848	0.000373785	0.001491487	0.000279259
-0.18	0.000192	0.000192	0.000413213	0.000418712	0.001664326	0.000313358
-0.2	0.000213333	0.000213333	0.000456963	0.000463005	0.001832366	0.000347173
-0.22	0.000234667	0.000234667	0.000500031	0.000506593	0.001995074	0.000380671
-0.24	0.000256	0.000256	0.000542348	0.000549406	0.002151916	0.000413821
-0.26	0.000277333	0.000277333	0.000175432	0.00017769	0.000690986	0.000134254
-0.28	0.000298667	0.000298667	0.000187637	0.000190027	0.000734061	0.000143982
-0.3	0.00032	0.00032	0.000199555	0.000202068	0.000774897	0.000153578
-0.32	0.000341333	0.000341333	0.000211165	0.000213792	0.000813333	0.000163031
-0.34	0.000362667	0.000362667	0.000222448	0.000225178	0.000849209	0.000172332
-0.36	0.000384	0.000384	0.000233383	0.000236205	0.000882366	0.000181472
-0.38	0.000405333	0.000405333	0.000243949	0.000246851	0.000912644	0.00019044
-0.4	0.000426667	0.000426667	0.000254125	0.000257095	0.000939882	0.000199228
-0.42	0.000448	0.000448	0.000263893	0.000266917	0.000963921	0.000207826
-0.44	0.000469333	0.000469333	0.00027323	0.000276295	0.000984601	0.000216225
-0.46	0.000490667	0.000490667	0.000282116	0.000285208	0.001001761	0.000224414
-0.48	0.000512	0.000512	0.000290532	0.000293635	0.001015243	0.000232385
-0.5	0.000533333	0.000533333	0.000298456	0.000301555	0.001024885	0.000240128

4.1.4 The results for transverse shear stress τ_{zx}^{EE}

The results for transverse shear stress τ_{zx}^{EE} are obtained from equation (3.2.23), equation (3.3.31), equation (3.4.46), equation (3.5.14), equation (3.6.14) and equation (3.7.14) of the ETB method, FSDT method, HSDT method, TSDT method, HPSDT method and V order method respectively in the Table 4.

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Results in Table 4. shows the variation of transverse shear stress τ_{zx}^{EE} through the thickness of simply supported Symmetrical I-Section beam from top $+h/2$ to bottom $-h/2$ when subjected to load of 20 KN/M

TABLE 4. transverse shear stress τ_{zx}^{EE} through thickness of the Beam from $+h/2$ to bottom $-h/2$

	ETB	FSDT	HSDT	TSDT	HPSDT	V ORDER
0.5	0	0	0	0	0	0
0.48	0.00000784	0.00000784	-3.39926E-06	-3.17719E-06	4.40314E-05	-7.11259E-06
0.46	0.00001536	0.00001536	6.91958E-06	7.78833E-06	0.000192325	-7.63554E-06
0.44	0.00002256	0.00002256	2.99233E-05	3.18303E-05	0.000436811	-2.04812E-06
0.42	0.00002944	0.00002944	6.44814E-05	6.77822E-05	0.00076866	9.12519E-06
0.4	0.000036	0.000036	0.000109384	0.000114396	0.001178424	2.53233E-05
0.38	0.00004224	0.00004224	0.000163361	0.00017036	0.001656179	4.59568E-05
0.36	0.00004816	0.00004816	0.000225097	0.000234317	0.002191652	7.04154E-05
0.34	0.00005376	0.00005376	0.000293248	0.00030488	0.002774346	9.80757E-05
0.32	0.00005904	0.00005904	0.000366456	0.000380649	0.003393662	0.000128308
0.3	0.000064	0.000064	0.000443365	0.000460224	0.004039006	0.000160484
0.28	0.00006864	0.00006864	0.000522635	0.00054222	0.004699898	0.00019398
0.26	0.00007296	0.00007296	0.000602949	0.000625281	0.005366071	0.000228187
0.24	0.000256533	0.000256533	0.002276775	0.002360304	0.020091874	0.000875041
0.22	0.0002688	0.0002688	0.002538867	0.002631283	0.022249334	0.000987962
0.2	0.00028	0.00028	0.002792221	0.002893201	0.02432895	0.001097582
0.18	0.000290133	0.000290133	0.003033188	0.00314229	0.026302205	0.001202209
0.16	0.0002992	0.0002992	0.003258376	0.003375051	0.028142625	0.00130027
0.14	0.0003072	0.0003072	0.003464683	0.003588284	0.029825974	0.001390325
0.12	0.000314133	0.000314133	0.003649316	0.003779107	0.031330446	0.00147108
0.1	0.00032	0.00032	0.003809815	0.003944979	0.032636815	0.001541392
0.08	0.0003248	0.0003248	0.003944067	0.004083721	0.03372858	0.001600281
0.06	0.000328533	0.000328533	0.004050323	0.004193527	0.03459208	0.001646935
0.04	0.0003312	0.0003312	0.004127208	0.00427298	0.035216591	0.001680719
0.02	0.0003328	0.0003328	0.004173737	0.004321062	0.035594398	0.001701173
0	0.000333333	0.000333333	0.004189312	0.004337158	0.035720851	0.001708022
-0.02	0.0003328	0.0003328	0.004173737	0.004321062	0.035594398	0.001701173

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-0.04	0.0003312	0.0003312	0.004127208	0.00427298	0.035216591	0.001680719
-0.06	0.000328533	0.000328533	0.004050323	0.004193527	0.03459208	0.001646935
-0.08	0.0003248	0.0003248	0.003944067	0.004083721	0.03372858	0.001600281
-0.1	0.00032	0.00032	0.003809815	0.003944979	0.032636815	0.001541392
-0.12	0.000314133	0.000314133	0.003649316	0.003779107	0.031330446	0.00147108
-0.14	0.0003072	0.0003072	0.003464683	0.003588284	0.029825974	0.001390325
-0.16	0.0002992	0.0002992	0.003258376	0.003375051	0.028142625	0.00130027
-0.18	0.000290133	0.000290133	0.003033188	0.00314229	0.026302205	0.001202209
-0.2	0.00028	0.00028	0.002792221	0.002893201	0.02432895	0.001097582
-0.22	0.0002688	0.0002688	0.002538867	0.002631283	0.022249334	0.000987962
-0.24	0.000256533	0.000256533	0.002276775	0.002360304	0.020091874	0.000875041
-0.26	0.00007296	0.00007296	0.000602949	0.000625281	0.005366071	0.000228187
-0.28	0.00006864	0.00006864	0.000522635	0.00054222	0.004699898	0.00019398
-0.3	0.000064	0.000064	0.000443365	0.000460224	0.004039006	0.000160484
-0.32	0.00005904	0.00005904	0.000366456	0.000380649	0.003393662	0.000128308
-0.34	0.00005376	0.00005376	0.000293248	0.00030488	0.002774346	9.80757E-05
-0.36	0.00004816	0.00004816	0.000225097	0.000234317	0.002191652	7.04154E-05
-0.38	0.00004224	0.00004224	0.000163361	0.00017036	0.001656179	4.59568E-05
-0.4	0.000036	0.000036	0.000109384	0.000114396	0.001178424	2.53233E-05
-0.42	0.00002944	0.00002944	6.44814E-05	6.77822E-05	0.00076866	9.12519E-06
-0.44	0.00002256	0.00002256	2.99233E-05	3.18303E-05	0.000436811	-2.04812E-06
-0.46	0.00001536	0.00001536	6.91958E-06	7.78833E-06	0.000192325	-7.63554E-06
-0.48	0.00000784	0.00000784	-3.39926E-06	-3.17719E-06	4.40314E-05	-7.11259E-06
-0.5	0	0	0	0	0	0

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4.2 Graphical representation of results of Symmetrical I-Section beam

4.2.1 The graph for axial displacement (u)

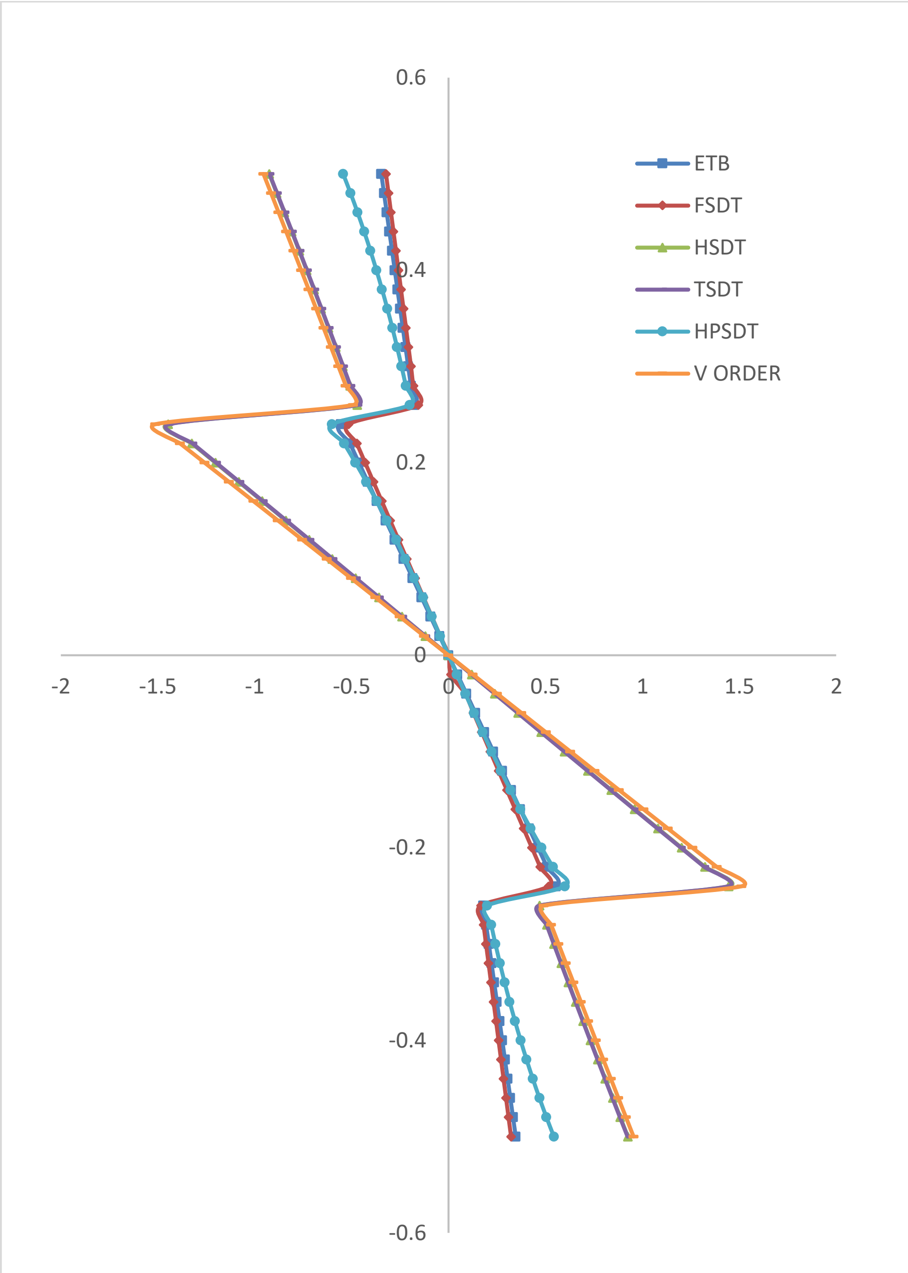

Fig 3. Variation of axial displacement ($\bar{u}$) through the thickness of Symmetrical I -beam from top $+h/2$ to bottom $-h/2$

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4.2.2 The graph for axial stress (σ_x)

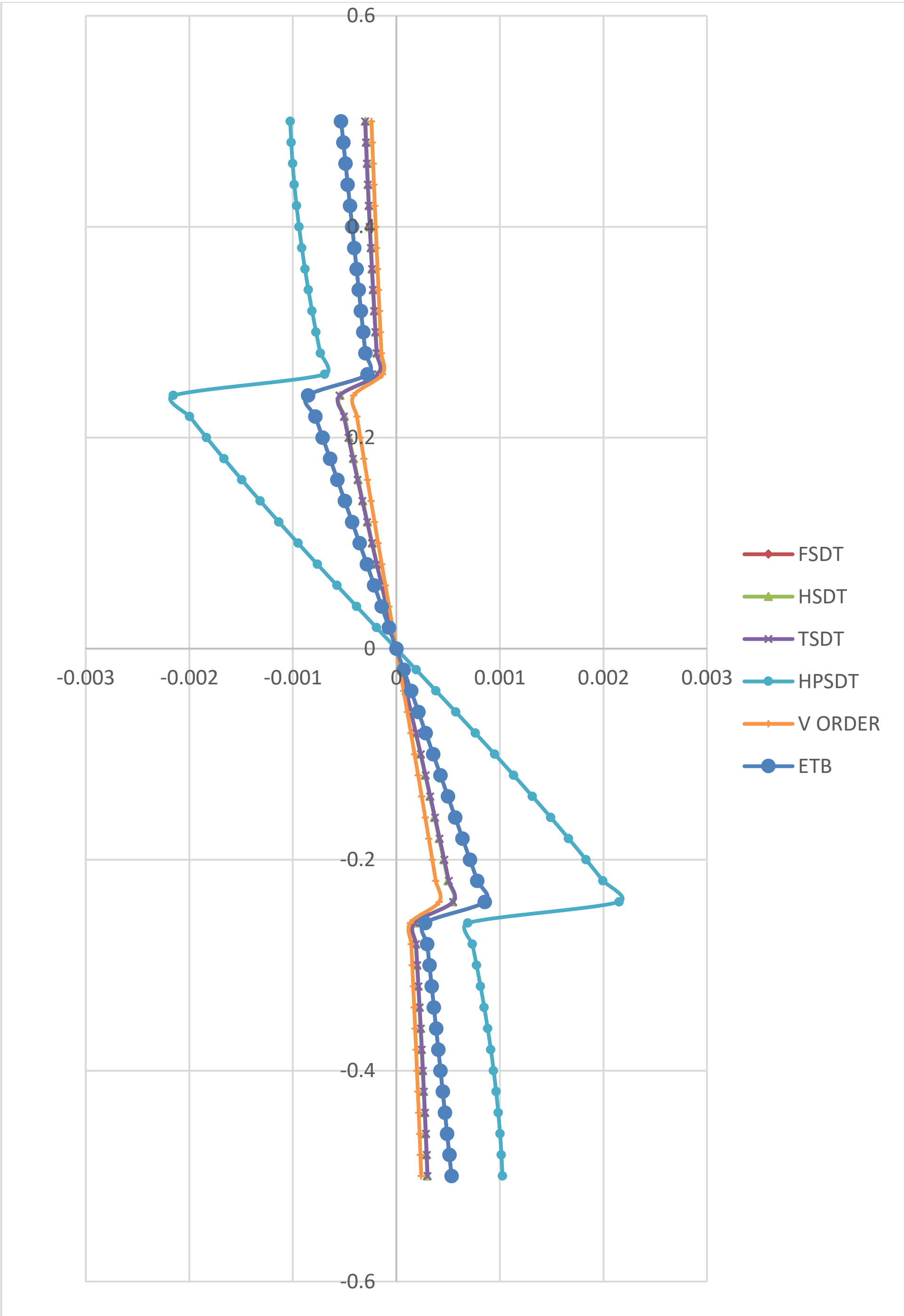

Fig 4. Variation of axial stress ($\bar{\sigma}_x$) through the thickness of Symmetrical I-Section beam from top $+h/2$ *to bottom* $-h/2$

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4.2.3 The graph for transverse shear stress τ_{zx}^{EE}

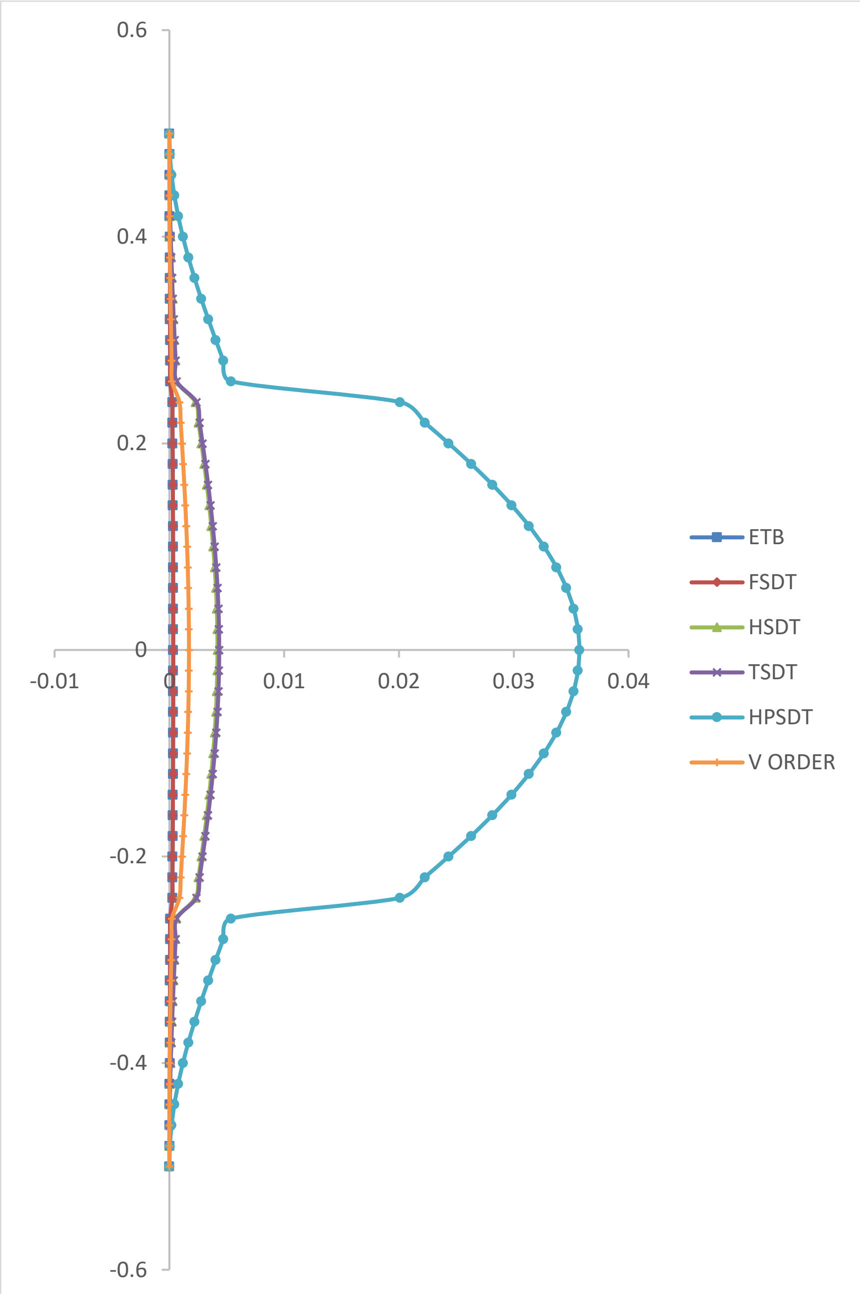

Fig 5. Variation of transverse shear stress ($\overline{\tau}_{zx}$) through the thickness of Symmetrical I-Section beam from top $+h/2$ to bottom $-h/2$

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5. INTERPRETATION OF RESULT

The results of the current fifth order shear deformation theory (V Order) are compared to those of the elementary beam theory (ETB/Classical), Timoshenko's first order shear deformation theory (FSDT), Krishna Murty's higher order shear deformation theory, and Ghugal and Sharma's hyperbolic shear deformation theory (HPSDT). Following is an interpretation of the results in this section.5.5.1 Interpretation of results of Symmetrical I- Section beam.

5.1 Interpretation of results of Symmetrical I Section Beam.

In Table No.2, the findings of Variation of axial displacement ($\bar{\sigma}_x$) are compared across the thickness of a simply supported beam from top $+h/2$ to bottom $-h/2$ under a load that varies evenly. All the revised theories' results and the axial displacement results provided by the current theory are closely related. When compared to other theories, the graphs for TSDT and V order reveal a little variance. In Table No.3, the comparison of the results of the fluctuation of axial stress along the thickness of a simply supported beam from top $+h/2$ to bottom $-h/2$ under a uniformly variable load is shown. Except for HPSDT, whose estimates of axial stress differ somewhat from those of the other theories, the results of the deformation theories are closely related. When a simply supported beam is exposed to a uniformly variable load, Table No. 4 compares the transverseshear stress results along the thickness of the beam from top $+h/2$ to bottom $-h/2$. Except for the larger scale of HPSDT values, the nature of the graphs created for each theory is comparable. In comparison to other theories, the results found for HPSDT for a given z/h value are higher.

6. CONCLUSION

From the results and discussion of present study following conclusions are drawn.

1. The transverse displacements, axial stresses and transverse shear stresses and their distribution obtained for all the theories show similar nature.
2. However, in few instances the values obtained for higher order theories like HPSDT and V order are more as compared to other deformation theories
3. In case of unsymmetric thick beam, for transverse shear stresses at top and bottom does not show zero stresses.

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