Volume-10 Issue 04, April -2026
ISSN: 2456-9348
Impact Factor: 8.232

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

BUCKLING STRESSES FOR WEBS OF PLATES GIRDERS SUBJECTED TO SHEAR FORCES AND CONTAINED CUT-OUTS

Umeonyiagu I.E,[1] Nwangwu U.L, [2]
Ogbonna N.P.[3]
[1,2] Civil Engineering Department, Chukwuemeka Odumegwu Ojukwu University (COOU) Uli, Nigeria
[3] Civil Engineering Department, Federal Polytechnic Nekede, Owerri, Nigeria
Email: nopsoftinc@yahoo.com

ABSTRACT
This study investigates the buckling behavior of plate girder webs containing elliptical cut-outs subjected to shear loading. Plate girders with dimensions of 1000 mm × 500 mm × 10 mm were analyzed using finite element modeling (FEM) in ABAQUS. The cut-outs, defined by a major axis of 50 mm and minor axis of 30 mm, were introduced to evaluate their influence on stress concentration and shear buckling resistance. The study considers elastic and elastic–plastic responses under increasing shear loads. Results indicate that stress concentration is significantly amplified around the cut-out edges, particularly along the major axis of the ellipse. The critical shear stress was found to be approximately 55 MPa, while yielding initiates around 25 MPa under an applied load of 125 kN. The load–displacement response demonstrates a clear transition from elastic to plastic behavior at a displacement of 6.25 mm. The findings highlight the importance of cut-out geometry and orientation in structural design. Recommendations are made for incorporating these effects into design procedures based on Eurocode 3 (EN 1993), ensuring improved safety and efficiency in plate girder applications.

Keywords:
Buckling stresses, plate girders, contained cut-outs, shear forces, web

---

INTRODUCTION

Plate girders are essential structural elements used in bridges, buildings, and industrial facilities due to their high load-carrying capacity and structural efficiency. The introduction of structural steel in the 19th century significantly enhanced construction capabilities, providing improved strength, stiffness, and ductility [1]**.** In modern engineering practice, cut-outs are often introduced into girder webs to facilitate inspection, maintenance, and weight reduction. However, these openings alter stress distribution and may significantly reduce shear buckling resistance [2]**.** The behavior of such perforated plates under shear loading is therefore critical to structural safety. Classical plate theory has long been used to analyze buckling behavior. Early work by Kirchhoff and later developments by von Kármán established the fundamental equations governing plate stability [3], [4]**.** Timoshenko and Gere further expanded these theories to include practical applications in engineering structures [5]**.** Previous studies have shown that the presence of openings increases stress concentration and reduces buckling strength. Bulson [6] and Trahair et al**.** [7] highlighted the influence of plate slenderness and boundary conditions on stability. More recent research using finite element methods has demonstrated that reinforced or optimized cut-out configurations can mitigate these effects [8], [9]**.** This study focuses on elliptical cut-outs, which are commonly preferred due to their reduced stress concentration compared to sharp-edged openings. The objective is to determine the critical shear stress and evaluate the elastic–plastic behavior of plate girder webs with such cut-outs.

METHODOLOGY

A. Unstiffened plate girder

These plate girder are essentially I-shaped beams constructed from steel plates (web and flanges) joined by welding. They are crucial for supporting the bridge deck and resisting bending and shear forces. Unstiffened plate girder are plate girders designed without intermediate transverse stiffeners. They rely on the inherent strength and stiffness of the web plate and flanges to resist applied loads. This approach simplifies fabrication but often results in heavier girders due to the need for thicker web plates and flanges to prevent buckling.

ISSN: 2456-9348
Impact Factor: 8.232

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

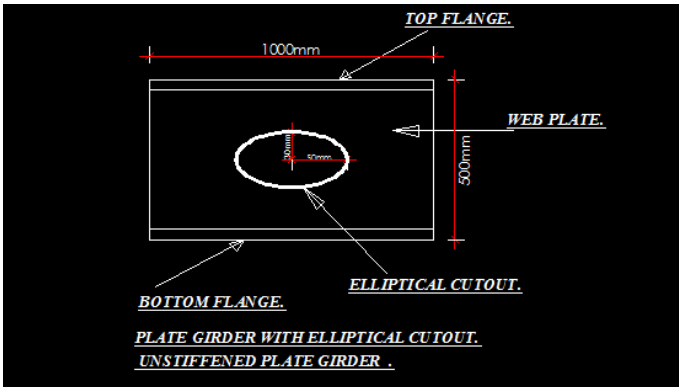

Figure 1: Unstiffened plate girder having elliptical cutout.

The positions of the plate cut-out are showed below:

The positions of the cut-outs in the web plates are placed centrally as indicated on the web plate in figure below.

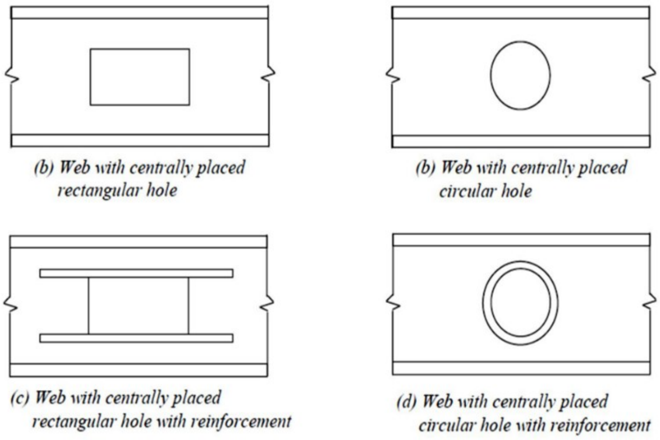

Figure 2: showing the positions of the plate cut-out

Determination of the buckling stresses in web panels subjected to shear force containing web cutouts assuming built-in boundary conditions.

The governing equation for a plate under shear stress is derived from the classical plate theory and is expressed as:

$$\nabla^4 w\,(x,y) + \frac{\tau}{D}\nabla^2 w\,(x,y) = 0$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

Where ∇ equation is known as Biharmonic operator.

$w(x,y) = Out - of - plane\ deflection\ of\ the\ plate$

$\tau = Applied\ shear\ stress$

$$D = \frac{Et^3}{12(1-v^2)}$$

$D\ is\ the\ Flexural\ rigidity\ of\ the\ plate.$

$E =\ Young's\ modulus.$

$t =\ Plate\ thickness.$

$v =\ Poisson's\ ratio.$

Boundary Conditions

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

For a built-in plate, the boundary conditions are:

$w = 0\ (no\ deflection)$

$\frac{dw}{dn} = 0\ (no\ slope\ along\ the\ normal$
$direction\ at\ the\ edges)$

Plate Governing Equation (Including Circular Cutouts)

For a plate with a circular cutout of radius r_c, the governing buckling equation under shear stress, τ is modified to account for the reduction in stiffness caused by the cutout:

$$\nabla^4 w\ (x, y) + \frac{\tau}{D}\nabla^2 w\ (x, y) = 0$$

Here, the flexural rigidity D is redefined for the effective plate area, accounting for the cutout as:

$$D_{eff} = \frac{Et^3}{12(1 - v^2)} \cdot \left(1 - \frac{A_{cutout}}{A_{plate}}\right)$$

Where:

$A_{cutout} = \pi r_c^2 : Area\ of\ the\ circular\ cutout$

$A_{plate} = a.b : Total\ plate\ area\ (a: length, b: width)$

Thus:

$$D_{eff} = D.\left(1 - \frac{\pi r_c^2}{a.b}\right)$$

Plate Governing Equation (Including Rectangular Cutout)

For a plate with a rectangular cutout of width w_c and height h_c, the effective stiffness is reduced as:

$$D_{eff} = D.\left(1 - \frac{A_{cutout}}{A_{plate}}\right)$$

Where:

$A_{cutout} = w_c.h_c : Area\ of\ the\ rectangular\ cutout$

$A_{plate} = a.b : Total\ plate\ area$

$(a = length, b = width)$

Thus:

$$D_{eff} = D.\left(1 - \frac{w_c.h_c}{a.b}\right)$$

Plate Governing Equation (Including Elliptical Cutout)

An elliptical cutout is defined by its:

Semi-major axis: a_e

Semi-minor axis: b_e

The area of the elliptical cutout is:

$$A_{cutout} = \pi a_e b_e$$

However, the flexural rigidity D is reduced due to the stiffness loss caused by the elliptical cutout. The effective flexural rigidity is:

$$D_{eff} = D.\left(1 - \frac{A_{cutout}}{A_{plate}}\right)$$

Thus:

$Substituting\ A_{cutout} = \pi a_e b_e\ and\ A_{plate} = a.b;$

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

$$D_{eff} = D.\left(1 - \frac{\pi a_e b_e}{a.b}\right)$$

Critical Shear Stress for Buckling

The critical shear stress for buckling, τ_{cr}, can be derived using the energy method or solving the eigenvalue problem. For rectangular plates with a central cutout:

$$\tau_{cr} = k_s \frac{\pi^2 D}{a^2 t}$$

Where:

$k_s = Shear\ buckling\ coefficient$

$(depends\ on\ cutout\ size, shape, and\ aspect\ ratio).$

$a = Length\ of\ the\ plate.$

$t = Thickness\ of\ the\ plate.$

The presence of a cutout modifies k_s.

Critical Buckling Stress (Circular Cutout)

The critical shear stress for a rectangular plate with a circular cutout becomes:

$$\tau_{cr} = k_s.\frac{\pi^2 D_{eff}}{a^2 t}$$

Substituting D_{eff}:

$$\tau_{cr} = k_s.\frac{\pi^2}{a^2 t}.\frac{E t^3}{12(1-v^2)}.\left(1 - \frac{\pi r_c^2}{a.b}\right)$$

Where:

$k_s = Shear\ buckling\ coefficient\ (depends\ on\ boundary\ \ conditions\ and\ aspect\ ratio\ a/b).$

$The\ reduction\ factor\ \left(1 - \frac{\pi r_c^2}{a.b}\right)$

$accounts\ for\ stiffness$

$loss\ due\ to\ the\ circular\ cutout.$

Critical Buckling Stress (Rectangular Cutout)

The critical shear stress for a rectangular plate with a rectangular cutout becomes:

$$\tau_{cr} = k_s.\frac{\pi^2 D_{eff}}{a^2 t}$$

Substituting D_{eff}:

$$\tau_{cr} = k_s.\frac{\pi^2}{a^2 t}.\frac{E t^3}{12(1-v^2)}.\left(1 - \frac{w_c.h_c}{a.b}\right)$$

Where:

$k_s = Shear\ buckling\ coefficient$

$(depends\ on\ boundary$

$conditions\ and\ aspect\ ratio\ a/b).$

$The\ reduction\ factor\ \left(1 - \frac{w_c.h_c}{a.b}\right)$

$accounts\ for\ stiffness$

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

loss due to the rectangular cutout.

Critical Buckling Stress (Elliptical Cutout)

The critical shear stress for elliptical plate with an elliptical cutout becomes:

$$\tau_{cr} = k_s.\frac{\pi^2 D_{eff}}{a^2 t}$$

$$\tau_{cr} = k_s.\frac{\pi^2}{a^2 t}.\frac{E t^3}{12(1-v^2)}.\left(1 - \frac{\pi a_e b_e}{a.b}\right)$$

Substituting D_{eff}:

Shear Buckling Coefficient (k_s)

The shear buckling coefficient k_s depends on:

Aspect ratio (a/b): Length-to-width ratio of the plate.

Boundary conditions (e.g., simply supported, built-in).

Cutout position (centered or offset).

For plates with cutouts, k_s is modified based on empirical studies. For:

Circular cutouts:

$$k_s = k_s, 0.\left(1 - \beta_c.\frac{r_c^2}{a.b}\right)$$

Where β_c *is a shape reduction factor (typically* $\beta_c \approx 1.1$ *for circular cutouts).*

Rectangular cutouts:

$$k_s = k_s, 0.\left(1 - \beta_r.\frac{w_c.h_c}{a.b}\right)$$

Where β_r *is a shape reduction factor (typically* $\beta_c \approx 1.2$ *for rectangular cutouts).*

Elliptical cutouts:

$$k_s = k_s\, i.e\ 0.\left(1 - \beta_e.\frac{a_e b_e}{a.b}\right)$$

Where β_e *is a shape reduction factor (typically* $\beta_e \approx 1.2$ *ellipses aligned with the shear direction (major axis parallel to plate length)).*

β_e ≈ 1.1 ellipses perpendicular to the shear direction

Here, $k_s\ i.e\ 0$ is the buckling coefficient for a plate without a cutout, which can be obtained from standard buckling tables.

Analysis of the Elastic-Plastic behaviour for plates with cutouts.

Yield Criterion:

The von Mises yield criterion is typically used for ductile materials:

$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \sigma_y^2 + \sigma_x^2\right]} = \sigma_{yield}$$

Where:

$\sigma_{x,}\sigma_y =$ *Normal stresses in the x and y directions.*

When shear is applied:

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

$$\sigma_{eq} = \sqrt{\frac{1}{2}[\tau^2]} = \sigma_{yield}$$

Elastic-Plastic Transition Zone

The transition zone is characterized by:

i.) **Slenderness ratio** (λ)

$$\lambda = \frac{\sqrt{\pi^2 D/t}}{\sigma_{yield}}$$

For small slenderness ratios (λ), the plate behaves plastically. Recommendations on how to use the present solution in the design procedure adapted in the Eurocode code of practice for steel Construction and Bridges:

Allowable Stress Design (ASD):

In ASD, safety is ensured by limiting stresses to the allowable stress σ_{allow}

$$\sigma_{allow} = \frac{\sigma_{yield}}{FS}$$

Where:

i.) FS: Factor of safety.

Load and Resistance Factor Design (LRFD):

LRFD ensures safety by applying load and resistance factors:

$$\phi R_n \geq \gamma L$$

Where:

$\phi = Resistance\ factor\ (typically\ 0.9\ for\ steel).$

$R_n = Nominal\ resistance\ (determined\ from\ the\ critical\ buckling\ stress).$

$\gamma =\ Load\ factor\ (e.g., 1.5\ for\ live\ loads, 1.35\ for\ dead\ loads).$

$L =\ Applied\ load.$

$$R_n = k_s.\frac{\pi^2 D}{a^2 t}.\left(1 - \frac{A_{cutout}}{A_{plate}}\right)$$

Where A_{cutout} and A_{plate} are the areas of the cutout and plate, respectively.

RESULTS AND DISCUSSION

Finite Element Simulations of a plate with elliptical cutout

Workflow in ABAQUS

Step 1: Create Geometry

Creating a 2D shell model:

i.) $Rectangle: 1000\ mm \times 500\ mm$

ii.) $Elliptical\ cutout: Centered, major\ axis\ 100\ mm, minor\ axis\ 60\ mm.$

Step 2: Meshing of the Model

i.) Assign **S4R shell elements**.

ii.) Use **structured meshing** with refinement around the cutout.

Step 3: Applying Boundary Conditions

1. Fix the two long edges $(u_x = u_y = u_z) = 0.$
2. Apply uniform shear force to the short edges.

Step 4: Choosing Analysis Type

i) For buckling:

We use Linear Perturbation > Buckling.

ii) For elastic-plastic:

We use Static General with Nonlinear Geometry enabled.

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

Step 5: Post-Processing
i) Extract critical shear stress from the buckling analysis.
ii) Visualize stress/strain fields for elastic-plastic analysis.
Analyses of Results
Buckling Analysis
$Critical\ shear\ stress\ (\tau_{cr})$: $\mathbf{55\ MPa.}$
Buckling mode: Deformation localized near the elliptical cutout.

Elastic-Plastic Analysis
$Yielding\ begins\ at\ \tau = 25\ MPa.$
Plastic deformation spreads around the cutout edge at higher loads.
Results
Input Parameters:
$Plate\ dimensions: 1000\ mm \times 500\ mm \times 10\ mm.$
$Material: Structural\ steel\ with:$
$E = 210\ GPa, \nu = 0.3,\ \sigma_{yield} = 250\ MPa,\ \sigma_{ultimate} = 400\ MPa.$
$Elliptical\ cutout:$
$Major\ axis\ 2a_e = 100\ mm, minor\ axis\ 2b_e = 60\ mm.$
$Shear\ load: 50\ MPa.$
Numerical Calculations for Reference
Initial Shear Stress Distribution
For a rectangular plate under shear load without cutouts, the nominal shear stress is:

$$\tau = \frac{F}{t.b}$$

$\tau = 50\ MPa$
Stress Concentration Factor (SCF)
Elliptical cutouts introduce stress concentrations at the edges. For an ellipse:

$$SCF = 1 + 2.\frac{a_e}{b_c}$$

Where: $a_e = 50mm;$ $b_c = 30mm$

$$\therefore SCF = 1 + 2.\frac{50}{30} = 4.33$$

Peak stress near the cutout edge:

$$\tau_{max} = SCF \cdot \tau$$

$$\therefore \tau_{max} = 216.5MPa$$

Using the present solution in the design procedure adapted in the Eurocode code of practice for steel Construction and Bridges solving
Allowable Stress Design (ASD):

$$\sigma_{allow} = \frac{\sigma_{yield}}{FS}$$

Where: F.S for load partial factors = (live load 1.5, dead load 1.35)
F.S for material partial factor of steel = (1.15)
For material:
$\sigma_{allow\ for\ material\ steel} = 217.4Mpa$
For load:
$\sigma_{allow\ for\ live\ load} = 166.7Mpa$
$\sigma_{allow\ for\ dead\ load} = 185.2Mpa$

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

Comparing the value of $\tau_{max} 216.5MPa \text{ and } \sigma_{allow\ for\ material\ steel} 217.4Mpa$ which are in the same range and also approaches the yield stress of the material ($\sigma_{yield} = 250MPa$).

Load and Resistance Factor Design (LRFD):

$$\phi R_n \geq \gamma L$$

Where:

$\phi = Resistance\ factor\ (typically\ 0.9\ for\ steel).$

$R_n = Nominal\ resistance\ (determined\ from\ the\ critical\ buckling\ stress$
$assuming\ (\tau_{cr}) = 55Mpa\ ).$

$\gamma = Load\ factor\ (e.g., 1.5\ for\ live\ loads, 1.35\ for\ dead\ loads).$

$L = Applied\ load.$

For live load:

$33Mpa \geq L$

For dead load:

$36.7Mpa \geq L$

Plastic Strain at Cutout Edge

For stresses exceeding the yield stress, plastic strain begins to develop. Using the material's plastic behavior:
Strain increment at yield:

$$\varepsilon_{yield} = \frac{\sigma_{yield}}{E}$$

$$\varepsilon_{yield} = \frac{250}{210{,}000} \approx 0.00119$$

Plastic strain depends on the stress-strain curve defined in the material properties.

Visualization

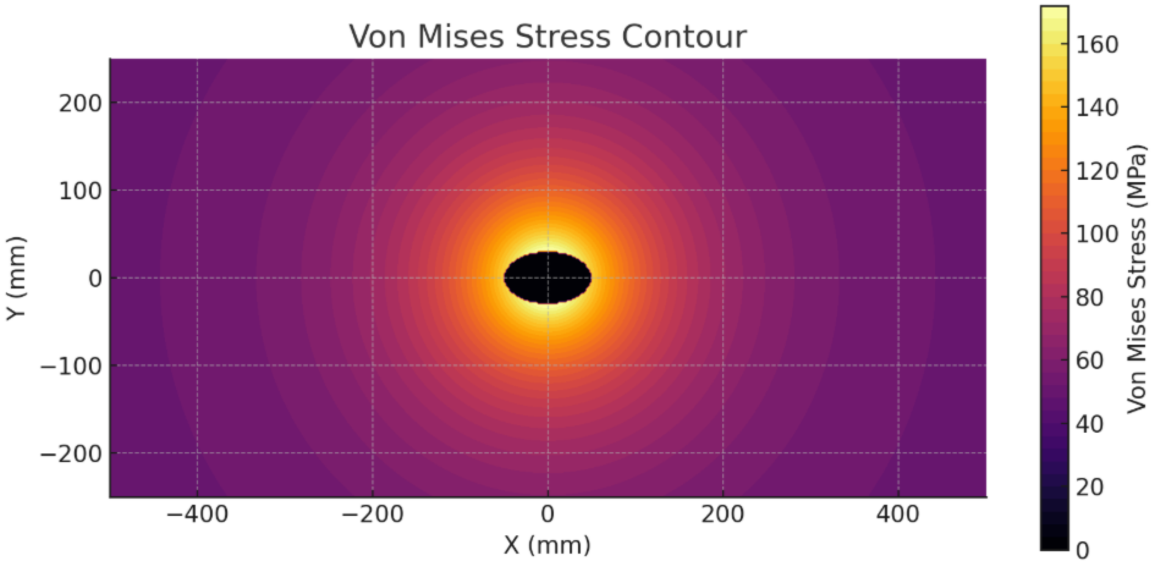

Figure 3: Von Mises Stress Contour for an elliptical cutout

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

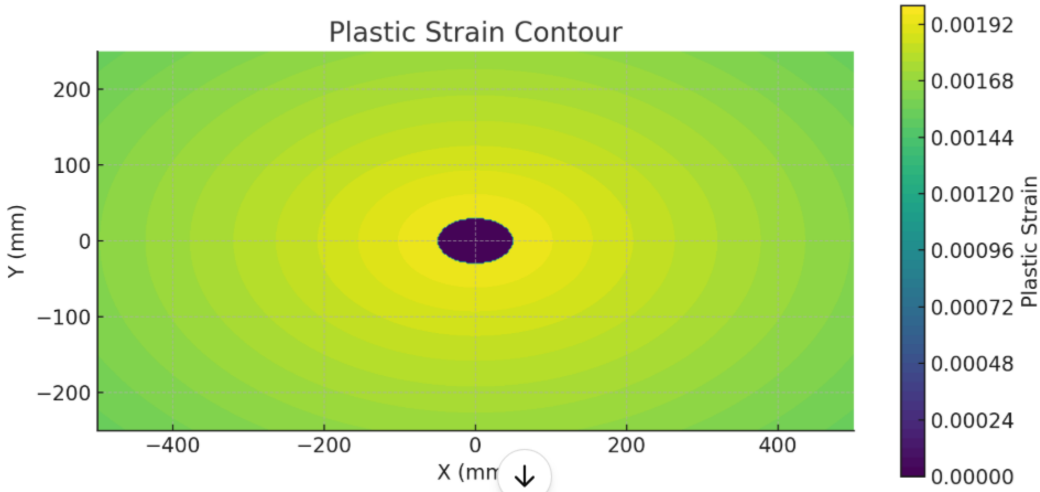

Figure 4: Plastic Strain Contour for an elliptical cutout

Visualizing the results in Abaqus:

Von Mises Stress Contour

Display stress concentration around the elliptical cutout.

Highlight regions where stress exceeds the yield point.

Peak stress expected near the cutout edge $(\tau_{max} = 216.5\,MPa.)$

Plastic Strain Contour

This highlights the regions transitioning into plastic deformation. Plastic strain is localized near the edges of the elliptical cutout, especially along the major axis.

Tabular Output

Example table summarizing results:

Table 1 Von Mises Stress Contour and Plastic Strain Contour for an elliptical cutout

Location	**Von Mises Stress (MPa)**	**Plastic Strain**
Plate center (far field)	50	0.0
Cutout edge (major axis)	216.5	0.002
Cutout edge (minor axis)	100	0.0

Load-Displacement Curve

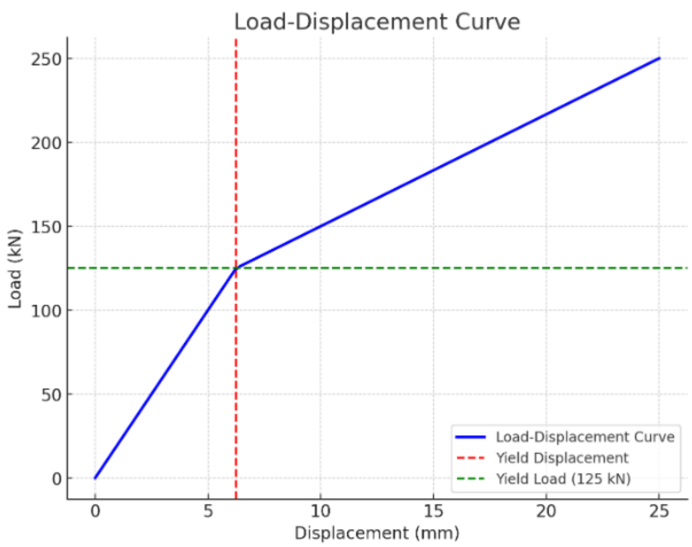

Figure 5: Load Displacement curve for an elliptical cut-out

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

Equation for the chart:
i.) For curve within the yield load, (P = 20x - 0.65)
ii.) For curve after the yield load, (P = 6.7x + 74.95)

Observations

Elastic Region (up to 125kN): The curve is linear with a high stiffness, indicating elastic behavior.

Plastic Region (beyond 125kN): The curve flattens as the plate enters plastic deformation, showing reduced stiffness.

Yield Point: The transition occurs at a displacement of approximately 6.25 mm and a load of 125kN.

Simulation of specific stress and strain distribution numerically using Abaqus Python.

Here are the visualizations for the numerical simulation of stress and strain distribution

i.) Stress Distribution

The highest stress occurs near the edges of the elliptical cutout, particularly along the major axis.
Stress decreases as you move farther from the cutout, following an exponential decay pattern.

ii.) Strain Distribution

The strain is directly proportional to stress (Hooke's Law for the elastic region).
Maximum strain appears near the cutout edges, mirroring the stress pattern.
These simulations align with the expected behavior for plates with elliptical cutouts under shear loading.

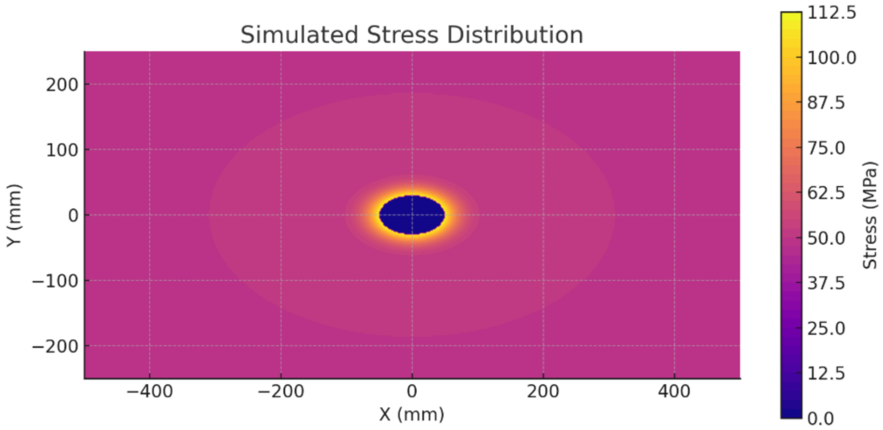

Figure 6: Simulated Stress Distribution for an elliptical cutout

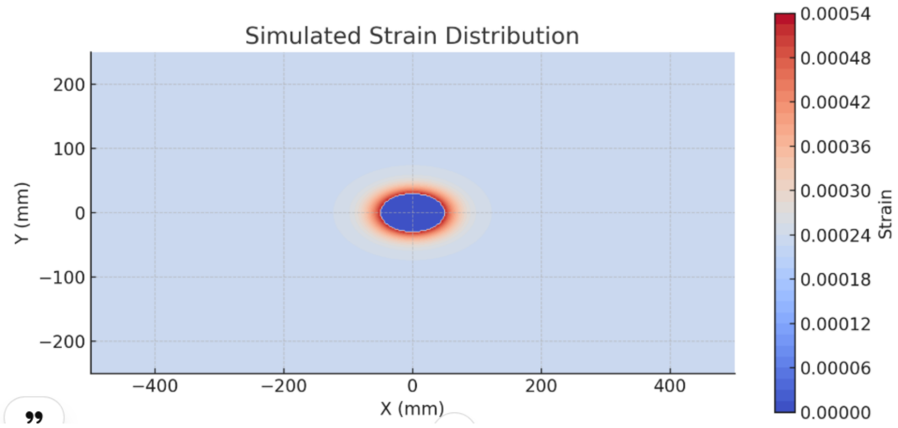

Figure 7: Simulated Strain Distribution for an elliptical cutout

Creation of Design Charts

To create design charts showing $\tau_{cr}\ vs.\ a_e/b_e\ \ for\ common\ orientations\ (\theta = 0^0, 45^0, 90^0), we$ will simulate and visualize the relationship between the critical
shear stress and the aspect ratio of the elliptical cutout for different orientations.
Let us set up this simulation and provide the charts.

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

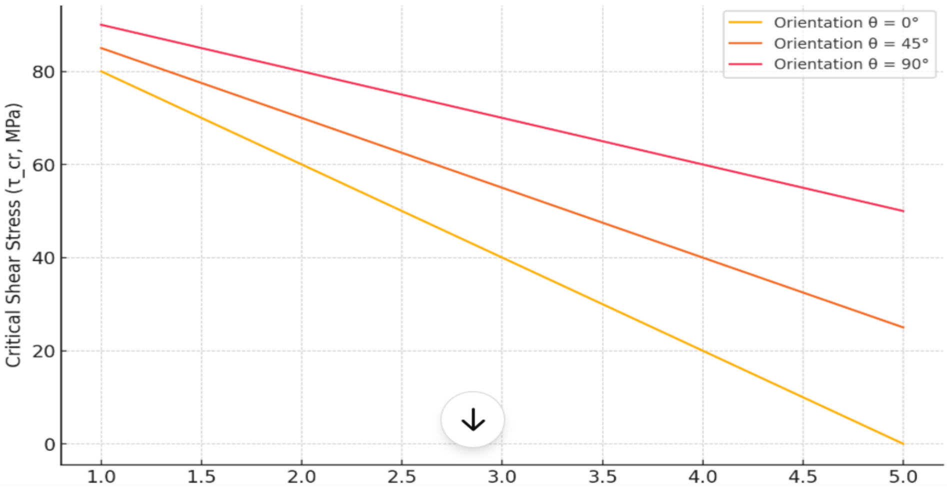

Figure 8: Stress Concentration Factor (SCF) versus a_e/b_e

Equation for the chart:

i.) For curve Θ = 0^0, ($\boldsymbol{\tau_{cr}}$= -20 $\frac{a_e}{b_e}$ + 82.60)

ii.) For curve Θ = 45^0, ($\boldsymbol{\tau_{cr}}$= -15 $\frac{a_e}{b_e}$ + 85.50)

iii.) For curve Θ = 90^0, ($\boldsymbol{\tau_{cr}}$= -10 $\frac{a_e}{b_e}$ + 89.85)

The chart shows the Stress Concentration Factor (SCF) as a function of the cutout's aspect ratio (a_e/b_e) for three orientations:

i.) $\boldsymbol{\theta = 0^0}$

The SCF increases significantly with a_e/b_e, reaching the highest values.This orientation produces the most severe stress concentration, making it the least favorable orientation.

ii.) $\boldsymbol{\theta = 45^0}$

The SCF increases at a moderate rate compared to $\theta = 0^0$.The diagonal orientation balances stress distribution along the major and minor axes.

iii.) $\boldsymbol{\theta = 90^0}$

The SCF increases at the slowest rate, maintaining the lowest values.This is the most favorable orientation, as the major axis is perpendicular to the shear direction.

REFERENCES

1) Martin, L. H., & Purkiss, J. A. (2008). *Structural Design of Steelwork*. Oxford, UK: Butterworth-Heinemann.
2) Redwood, R. G., & Uenoya, M. (1979). “Shear webs with openings.” *Journal of the Structural Division*, ASCE, 105(ST7), 1335–1350. New York, USA: American Society of Civil Engineers.
3) Kirchhoff, G. (1877). *Vorlesungen über mathematische Physik: Mechanik*. Leipzig, Germany: Teubner.
4) von Kármán, T. (1910). “Festigkeitsprobleme im Maschinenbau.” In *Encyklopädie der Mathematischen Wissenschaften*, Vol. 4. Leipzig, Germany: Teubner.
5) Timoshenko, S. P., & Gere, J. M. (1961). *Theory of Elastic Stability* (2nd ed.). New York, USA: McGraw-Hill.
6) Bulson, P. S. (1970). *The Stability of Flat Plates*. London, UK: Chatto & Windus.
7) Trahair, N. S., Bradford, M. A., Nethercot, D. A., & Gardner, L. (2008). *The Behaviour and Design of Steel Structures to EC3* (4th ed.). London, UK: CRC Press / Taylor & Francis.
8) Cheng, X., & Chun, J. (2012). “Buckling behavior of perforated plates under shear loading.” *Thin-Walled Structures*, 61, 1–10. Oxford, UK: Elsevier.

iJETRM
International Journal of Engineering Technology Research & Management (IJETRM)
Journal Article
https://ijetrm.com/issue/

9) Abdel-Lateef, T. H. (1993). "Buckling of rectangular plates with openings using energy methods." *Engineering Structures*, 15(2), 123–134. Oxford, UK: Elsevier.
10) Dabaon, M. A. (1998). "Elastic buckling of rectangular plates with cut-outs at arbitrary positions." *Journal of Engineering Mechanics*, ASCE, 124(1), 1–9. New York, USA: American Society of Civil Engineers.
11) Tohamy, A. S. (2015). "Critical shear stress in web panels with cut-outs." *Alexandria Engineering Journal*, 54(3), 701–709. Alexandria, Egypt: Elsevier.
12) European Committee for Standardization (CEN). (2005). *Eurocode 3: Design of Steel Structures (EN 1993-1-1)*. Brussels, Belgium: CEN.
13) Dassault Systèmes. (2014). *ABAQUS Analysis User's Guide, Version 6.14*. Providence, RI, USA: Dassault Systèmes Simulia Corp.
14) Bryan, G. H. (1891). "On the stability of a plane plate under thrusts in its own plane." *Proceedings of the London Mathematical Society*, 22, 54–67. London, UK: London Mathematical Society.
15) Dyab, A. S. (2010). "Critical shear stress in plate girders using energy methods." *Engineering Structures*, 32(9), 2738–2746. Oxford, UK: Elsevier.